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Equation with  
Oscillating Charge



## 2.1 Heuristic Premises and “Derivaton” of the Equation with Oscillating Charge for a Single Particle; Theorem on “Hidden” Parameters

*I think it is quite possible that in the long run Einstein was right that the current form of quantum theory should not be considered as final... I think it is quite possible that in the future a new improved quantum mechanics appears containing a return to the determinism and confirming Einstein's point of view. But such return to the determinism is possible only at the cost of rejection of some fundamental ideas, which we accept now without any doubts. If we return to the determinism we shall have to pay for it in some way, although it is difficult to foresee how.*

*P. A. M. Dirac, 1978 [164]*

In the sect.1.4 we have found the solution of the simplified scalar integro-differential equation resulted in the localized solution for the wave packet/particle. It appeared that the integral of bi-linear combination of such solution over the whole volume is equal to the value of the dimensionless elementary electric charge with the precision up to 0.3% [7, 8]. As we tried to solve the problem in the form of a periodically appearing and vanishing wave packet it was easy to associate such solution with simple space electric charge oscillation that has a double-charge amplitude. Later that packet may be replaced by an oscillating point charge. This movement may be described by the general Newton equation. But we should take into consideration the changes of point' characteristics within process of movement. In the essence, it is simply the next

step in the theory of material point's motion. It is not a new idea for ordinary mechanics. There are well known equations of I. Metchersky for the motion of variable mass bodies and K. E. Ciolkowski equations for rockets motion. But until now according to standard quantum theory, a particle has the fixed set of characteristics in space-time field.

It is important to note that Sir Isaac Newton did not apply the conception of material point at all, although it is ridiculous to imagine that such a natural and trivial idea could not come into his mind. We do not know the way of thinking of that great man. But we know about his marvellous and it may be quite probable that Newton felt and foresaw intuitively all difficulties which the physics science should meet when using the conception of a material point, and he wanted to warn the physicists of future generations: "Be careful! The notion of a material point is dangerous!"

Really, we see today- after more then two and a half century - that the biggest troubles of the quantum theory arise if a particle is considered as a material point. A rich bouquet of divergences is the result of this approach. Nevertheless, such an approach is very convenient if it should be used correctly. Let us remember that in accordance with the Newton corpuscular theory, beams of light should be considered as a flow of certain particles. They are emitted in all directions by a luminous body and move in empty space or homogeneous medium uniformly and linearly. In other words, in the same way as usual ordinary material particles do in the absence of any external forces. Newton explained the phenomena of reflection and refraction of light beams on the interface between two homogeneous mediums as a result of the certain forces action directed orthogonally to this interface. These forces, according to Newton, change the normal velocity component, but do not touch the tangential one, and the analysis of this effect has allowed to derive the laws of reflection and refraction. However,

the inability of his theory to explain the effects of partial reflection and passage phenomena as well as Newton rings (his own discovery) brought him to almost forgotten but quite modern today theory of bouts (fits). Newton thought that to make complete explanation of all the processes it was necessary to assume that particles of light may experience bouts of reflection and bouts of passage as well. Assume the light falling on to a flat surface. Some part of beams passes and other is reflected. Following quantum description of that effect the particle connected with the incident wave at the moment of impact has a certain probability to pass or to be reflected. In this situation Newton just used the word “bouts” instead of “probability”.

It is absolutely clear that ideas set forth below will be crude approximation, because no one equation of particle’s motion is able to describe even the most simple interference process in the case of translucent mirror. During that process material particle is divided into two parts, that later shall destroy each other in destructive interference. If we would like to make correct description of single particle, then situation from viewpoint of standard quantum mechanics becomes dismal and purely probabilistic. At any moment of time a particle may be in only one non-coherent state: no one particle can move in two different directions simultaneously. Nevertheless, it seems there is a whole class of processes where such description has certain sense.

The equation with oscillating charge was derived soon after the thin structure constant value estimation was obtained. For the first time this equation was just postulated [53, 54, 172, 183] and used for description of cold nuclear fusion process due to mutual deuteron interaction (see sect.3.1).

This equation has the following form

$$m \frac{d^2 \mathbf{r}}{dt^2} = -2Q \text{grad } U(\mathbf{r}) \cos^2 \left( \frac{m\mathbf{t}}{2\hbar} \left( \frac{d\mathbf{r}}{dt} \right)^2 - \frac{m\mathbf{r}}{\hbar} \frac{d\mathbf{r}}{dt} + \phi_0 \right), \quad (2.1.1)$$

where  $m$  is the mass,  $\mathbf{r}$  the radius vector,  $U(r)$  the external potential, the initial phase and  $Q$  the constant part of particle's charge.

As soon as  $\mathbf{E} = -\text{grad}U$ , and there exists a magnetic field for every electro-magnetic field one should take into account the Lorentz force  $\mathbf{F} = \frac{Q}{c} [\mathbf{v} \times \mathbf{H}]$ . In electromagnetic mode  $\mathbf{E}$  and  $\mathbf{H}$  are similar, for small energies value  $\frac{v}{c} \rightarrow 0$  and force  $\mathbf{F}$  may be neglected.

The multiplicator 2 in (2.1.1) is needed for correct transition to equation of classical mechanics because the averaged charge will be two times smaller.

The transition from quantum mechanics to classical mechanics is usually rigorous and overcome many difficulties. But there are some serious problems. For example, the conception of spin has led to some problem. G. Uhlenbeck and S. Goudsmit propose the notion of spin in 1925 after analysis of spectroscopic data. In order to explain these data it was necessary to suppose the existence of eigen mechanics momentum  $\frac{\hbar}{2}$  and connected with it magnetic momentum equal to Bohr magneton  $\mu_B = \frac{e\hbar}{2mc}$ . The same quantities were obtained later by Dirac's equation. Then the ratio of spin magnetic momentum to mechanics momentum equals  $\gamma = \frac{e}{mc}$ . This value of  $\gamma$  is anomal and should be two times smaller because in the case of orbital motion of electron and of any classics

system motion of charged particles' system have given ratio  $\frac{e}{m}$  equal to  $\frac{e}{2mc}$ .

The reason of such disaccord is not explained but we understand now that it is connected with oscillation and averaging of charge. Any problem disappear after accepting this fact.

Besides, there is another anomaly not mentioned before in physical literature and connected with kinetic energy of particle. Let the particle with mass  $m$  and moving with velocity  $V$  to possess the de Broglie wave length  $\lambda$  and energy  $E$  equal to

$$\lambda = \frac{h}{mV}, \quad E = \hbar\omega,$$

where  $\hbar = \frac{h}{2\pi}$ . Then the particle with velocity  $V$  will pass the way equal  $\lambda$  during interval of time equal to  $T$  (period):

$$T = \frac{\lambda}{V} = \frac{h}{mV^2} \quad v = \frac{1}{T}$$

It is possible now to find the energy of particle:

$$E = \hbar\omega = hv = mV^2,$$

This value is two times more than ordinary value of kinetic energy. So, the averaging of oscillations process causes the above-mentioned disaccord.

And still great dissatisfaction remains because equation (2.1.1) is only postulated. More over, the fact that not every particle is charged strictly restricte the use of equation. A little bit later [55-58] that equation was "derived" from Schroedinger equation it was understood the specific character of charge

oscillation. However, for more simplicity we are going to use “oscillating charge” term. It was H. Poincare [161] who noticed for the first time that if the charge or mass of the particle were equally decreased it would not influence equations of motion and could not be experimentally detected.

Let us notice at the same moment that quantum mechanics is the more fundamental science than classical mechanics. As it approaches the limit quantum mechanics results in classical mechanics. However, that fact had not prevented Schroedinger to “deriving” his “famous” equation from relations obtained within Newton mechanics. Schroedinger himself (and many other researchers) considered it not as rigorous deduction but a peculiar illustration because it was impossible to derive this equation strictly from classical mechanics, and this equation was, in fact, postulated. Quite similarly, the equation with oscillating charge is not contained in Schroedinger equation, and further we propose some illustration of correspondence between these two equations.

We will “derive” equation (2.1.1) from Schroedinger equation in the following way [200, 201]. Let us do it for one-dimensional case, since 3-dimensional generalization is too complicated.

Complete Schroedinger equation with potential  $U(x)$  is following:

$$\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t) + i\hbar \frac{\partial}{\partial t} \Psi(x, t) = U(x) \Psi(x, t) \quad (2.1.2)$$

We will seek the solution of this equation in non-traditional form:

$$\Psi(x, t) = \cos(kx) \int \exp(itg(\phi)) dt, \quad (2.1.3)$$

where

$$\phi = \frac{mt}{2\hbar} \left( \frac{dx(t)}{dt} \right)^2 - \frac{mx(t)}{\hbar} \frac{dx(t)}{dt} + \phi_0 \quad (2.1.4)$$

The  $x(t)$  function is some function of time and is not connected in any way with independent variable  $x$ . By substituting (2.1.3) in equation (2.1.2) we get:

$$i\hbar^2 k^2 \int \exp(itg(\phi)) dt + 2imU(x) \int \exp(itg(\phi)) dt + 2\hbar m \exp(itg(\phi)) = 0 \quad (2.1.5)$$

For the very small kinetic energies the following relation always holds true:

$$\hbar^2 k^2 \ll 2mU(x).$$

Then we may neglect the first integral in (2.1.5). Differentiating the remnant part in time and reducing general exponential factor we obtain:

$$2U(x) \cos^2(\phi) + 2mt \frac{dx(t)}{dt} \frac{d^2x(t)}{dt^2} - m \left( \frac{dx(t)}{dt} \right)^2 - 2mx(t) \frac{d^2x(t)}{dt^2} = 0 \quad (2.1.6)$$

If we use the relation

$$x(t) \approx t \frac{dx(t)}{dt},$$

that may be considered true for short time-intervals, then in equation (2.1.6) items 2 and 4 are canceled and we obtain:

$$U(x) \cos^2(\phi) = \frac{m}{2} \left( \frac{dx(t)}{dt} \right)^2 \quad (2.1.7)$$

In the equation (2.1.7) left side is oscillating potential energy, right is kinetic energy. Unfortunately, we do not observe mutual transformation of kinetic energy into potential one and back (as it is in classical mechanics of different conservative systems). It seems that potential energy oscillate because the whole packet appears and disappears together with the charge. At the other side, kinetic

energy apparently is connected with Fourier harmonic components of moving packet that results in appearance and disappearance of mass due to dispersion in the process of moving. Then we shall assume that independent variable  $x$  should be replaced by  $x(t)$  in the potential; we have no other simple idea. In that case we get the following equation:

$$U(x(t)) \cos^2(\phi) = \frac{m}{2} \left( \frac{dx(t)}{dt} \right)^2, \tag{2.1.8}$$

which may be considered as typical Lagrangian like:

$$L = \frac{m}{2} (\dot{x})^2 - U \cos^2(\phi), \tag{2.1.9}$$

where  $x$  depends on time and following shorter symbols are used:

$$x = x(t), \dot{x} = \frac{dx(t)}{dt}, \ddot{x} = \frac{d^2x(t)}{dt^2}, U(x) = U$$

If we integrate that Lagrangian, then we obtain the expression for the action. Further, we can find Euler-Lagrange equation; that will be the equation of motion. For this purpose we integrate Lagrangian (2.1.9) in respect to time and obtain the action functional, and compile the variation of this functional. We get the equation:

$$\begin{aligned} & -U \dot{x} \cos^2(\phi) + \frac{2}{\hbar} U m \dot{x} \cos(\phi) \sin(\phi) - m \ddot{x} - 2U \dot{x} \cos(\phi) \sin(\phi) \left( \frac{m \dot{x} t - m x}{\hbar} \right) \\ & + 2U \sin^2(\phi) \left( \frac{m \dot{x} t \ddot{x} - m \ddot{x} x}{\hbar} - \frac{m \dot{x}^2}{2\hbar} \right) \left( \frac{m \dot{x} t - m x}{\hbar} \right) \\ & - 2U \cos^2(\phi) \left( \frac{m \dot{x} t \ddot{x} - m \ddot{x} x}{\hbar} - \frac{m \dot{x}^2}{2\hbar} \right) \left( \frac{m \dot{x} t - m x}{\hbar} \right) \\ & - \frac{2}{\hbar} U m \ddot{x} \cos(\phi) \sin(\phi) = 0 \end{aligned} \tag{2.1.10}$$

If we agreed that within infinitesimal time interval the velocity and acceleration of particle are nearly constant, i.e.

$$x \approx x't, x' \approx x''t,$$

then only the first and the third items remain. Thus, we can rewrite (2.1.10) in habitual form:

$$\frac{dU(x)}{dx} \cos^2(\phi) = m \frac{d^2x(t)}{dt^2} \quad (2.1.11)$$

In 3-dimensional case we obtain the same result. Notice, equation (2.1.11) is non-autonomous according to expression (2.1.4) for  $\phi$ .

Our autonomous equation in form (2.1.11) can “be derived” from Schroedinger equation also. For this purpose we will seek the solution of equation (2.1.2) in form (2.1.3), but with another phase:

$$\phi = \frac{mc^2t}{\hbar} + \frac{mt}{2\hbar} \left( \frac{dx(t)}{dt} \right)^2 - \frac{mx(t)}{\hbar} \frac{dx(t)}{dt} + \phi_0. \quad (2.1.12)$$

Then after substitution at Shroedinger equation and after the same transformations as previous one we will get new first-order equation called Lagrangian:

$$U(x(t)) \cos^2(\phi) - \frac{m}{2} \left( \frac{dx(t)}{dt} \right)^2 + mc^2 = 0. \quad (2.1.13)$$

After integrating (2.1.13) in respect to time and compiling the variation we get equation in form (2.1.11), but with the phase in form (2.1.12). In expression (2.1.12) there are terms with slow and fast oscillation that satisfy following inequality:

$$\frac{mc^2}{\hbar} \gg \frac{m}{2\hbar} \left( \frac{dx(t)}{dt} \right)^2$$

Now we may first of all neglect the smaller term in comparing to the bigger one, and then reject fast oscillating term, as far as it has no influence on final result. Thus we have the autonomous equation that may be written as follows:

$$m \frac{d^2r}{dt^2} = -Q \text{grad}U(\mathbf{r}) \cos^2\left(-\frac{mr}{\hbar} \frac{dr}{dt} + \phi_0\right) \quad (2.1.14)$$

Of course, this method doesn't delight anybody, but it differs a little from generally accepted cancellation of divergences in quantum field theory, when infinities being subtracted one from the other are canceled.

It should be noticed that autonomous equation (2.1.14) may be obtained after substituting relations (2.1.10) into (2.1.4). It should be especially underlined that resulted first-order equations like (2.1.8) and (2.1.13) won't be primary integrals of the second-order equations (2.1.1) and (2.1.14) and last equations are crude approximations. More over the entire "derivation" may be a subjected to criticism. Our main task, however, is to illustrate that the above-mentioned equations have certain relation with Schroedinger equation. By the way, in "hidden parameters" theorem it was logically proven that within rigorous Schroedinger equation there was no place for such hidden parameter as initial phase. That is why the rigorous deduction of our equations from Schroedinger equation is absolutely impossible. Hereafter we will try to explain how it should be understood at all.

We deal in quantum theory with pure probabilities and such approach is based not upon our inability to control or exactly measure different parameters of the existing processes, but upon accidental character of many parameters by its nature. In other words, the chance that observed probability reflects the influence of uncontrolled hidden parameters may be excluded from consideration, if these

parameters are not clearly detected or are not included into theory. According to that quantum mechanics assume that alternative events have equal probabilities and consider it as a physical fact. More generally, it is considered as a basic thesis limited reproduction of the atomic events to be occurred in thoroughly controlled similar experimental conditions.

The main aim of the Science is the understanding of outward things and description of all going processes by means of Mathematics. One way is gaining experimental information and putting it in good order in our mind. That process requires considering as fundamental or initial some minimal quantity of facts, and the other facts as their logical corollaries. Such division into fundamental facts and their logical corollaries depends on analytical abilities as well as on existing in Science of an overarching paradigm and some times on our preferences as well. For example, it is unnecessary to consider mathematical beauty of a theory as truth criterion (P. A. M. Dirac). As alternative example we can use Lorentz fundamental transformations (at our point of view they are quite not good-looking) or Maxwell equations, which beauty till introduction of mono-field (P. A. M. Dirac again) was rather doubtful.

Newton mechanics, uniquely, allows the prediction (in a determined way) the future of a system if the initial data is known. Statistic mechanics arises from the necessity of complicated mechanical systems' analysis, when small and even uncontrollable inexactitude of initial data results in almost unforeseen consequences and so makes concessions to very complicated computational processes. Nevertheless, determined process remain the base of statistical mechanics.

Within standard quantum mechanics the situation absolutely differs. According to it, dynamics and statistics are indivisible, and not even the most genius mathematician with the most powerful super computer principally can

avoid a statistical description. And here an atavistic thought appears that in reality quantum description is incomplete, and in future, when new “hidden” parameters yet unknown for quantum mechanics will be introduced, the descriptions of predicted determined dynamic regularity may arise. For the first time that challenge was strictly issued and solved by mathematician John von Neumann for the Schroedinger equation: there were no such “hidden” parameters in standard quantum theory with Schroedinger equation.

The equation with oscillating charge has such “hidden” parameter – the initial phase. Naturally, the question arise, how to reconcile it with von Neumann proof. Here we can notice that equation with oscillating charge is a crude approximation at very small energies and therefore, formally, it is not strict quantum-mechanical equation and results of von Neuman theorem [170] can not be applied.

It is quite understandable that equation with oscillating charge can not strictly describe interference processes since according to it moving particle should have bifurcation’s states (particle should physically divide). This is absolutely impossible in the case of motion equations in classical mechanics. That is why using our equation is apparently limited to cases of small energies and cases when there is evidently no interference or strong diffraction. In other words, in the case, when the wave packet is being reflected or dispersed as a whole only, then the use of equation with oscillating charge is possible.

Moreover, according to such approach the question about particle’ photon emission when particle starts moving with acceleration remains unclear. Generally speaking, intimate mechanism of photon emission remains a big mystery. We assume the picture of such a process in images and movements exists and we hope it will be discovered in future.

There is very interesting parallel between Schroedinger equation and equation

with oscillating charges. It is known that in the case of charged particle movement in plane condenser with the constant tension to be applied classical uniformly accelerated motion  $x = at^2$  appears. For the equation with oscillating charge such analytical solution exists (see sect. 2.2-2.4). Let show that Schroedinger equation has physically similar solution also. Viz., let potential in Schroedinger equation be equal to  $U(x) = rx$ . Then complete Schroedinger equation as follows:

$$\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} - rx\Psi + i\hbar \frac{\partial \Psi(x, t)}{\partial t} = 0 \quad (2.1.15)$$

We will seek the solution in rather unusual form:

$$\Psi(x, t) = b \exp\left(i \frac{ma^2 t^3}{2\hbar} - i \frac{matx}{\hbar}\right) \quad (2.1.16)$$

By substituting (2.1.16) in (2.1.15) we get (after reducing):

$$-2ma^2 t^2 + (ma - r)x = 0.$$

This relation will be fulfilled if

$$x = \frac{2ma^2}{ma - r} t^2. \quad (2.1.17)$$

This result confuse untrained reader, because in equation (2.1.15)  $x$  and  $t$  are independent from each other variables. Such idealization is inherent and convenient in mathematics, but the real situation is slightly others: during motion the truly independent variable is time only. Generally speaking, coordinate is dependent variable and at given velocity is connected with time by means of the relation (2.1.17).

If in (2.1.17) impose the requirement  $r \rightarrow 0$  (potential vanishes), then absolutely strange particular solution appears where the particle is able to move

with constant acceleration and to generate energy of an unknown origin (!!!) Of course, it is out of understanding how such initial conditions could be created. That effect remains valid even if we put  $r \rightarrow 0$  directly in equation (2.1.15).

From the point of view of standard physics the motion of quantum particle within the field of constant potential never differs from the motion in empty space free from any field, because, as a rule, potential is determined up to arbitrary constant (well known calibration) and that constant may be always selected so as potential would be equal zero. Such a solution of the equation (2.1.15) for wave function with increasing frequency (energy) has been discovered independent from us by Dr. Bill Page - USA (particular report) in the form of combinations of Airy functions. The same solutions can be obtained for Dirac equation.

Curious, but we have similar situation in classical electrodynamics. If during acceleration of a charge one takes into account force acting on a charge itself, then the braking due to radiation arises. In different works this effect is called in different way: bremsstrahlung, Lorenz frictional force or Plank's radiant friction. That force is proportional to third derivative of coordinate  $x$  relative to time and is experimentally proved many years ago. If we write the equations of motion for the charge moving in space free from impact of external fields and if the only force acting on the charge is the "Plank radiant friction", then we will obtain the following equation:

$$m \frac{d^2 x}{dt^2} = \frac{2e^2}{3c^3} \frac{d^3 x}{dt^3}$$

It is evident that equation in addition to trivial and natural particular solution

$$v = \frac{dx}{dt} = \text{Const}$$

has general solution where particle acceleration is equal

$$a = \frac{d^2 x}{dt^2} = C_1 \exp\left(\frac{3mc^3 t}{2e^2}\right),$$

i.e. it is not only unequal to zero, but more over it unrestrictedly exponentially increases in time for no reason whatever!!! For example, L. Landau and E. Lifshits in their classical work “Theory of the field” wrote apropos of this: “*A question may arise how electrodynamics satisfying energy conservation law is able to give rise to such an absurd result in accordance to which a particle was able to unrestrictedly increase its energy. The background of that trouble is, actually, in infinite electromagnetic “eigen mass” of elementary particles. If we write in equations of motion finite charge mass, then we, in essence, arrogate to it formally an infinite, formally, negative “eigen mass” of not electro-magnetic origin that together with electro-magnetic mass should result in finite mass of particle. But as far as subtraction of one infinity from another is not mathematically correct, that leads to troubles as described above*”.

We are going to tell about such astonishing solutions, where excess energy appears in further chapters of our book. We think that processes of energy generation in nature have left their signs both in quantum theory and electrodynamics. We should note that such traces are fully absent in classical mechanics.

## 2.2 Autonomous Equations and Some Their Properties

*To learn a secret one needs much to guess*

A. S. Griboyedov

The vector equation of the motion of charged particle in the field under some forces has in our autonomous model general form (2.1.14). Let us write that equation in simplified form assuming that  $m = 1, \hbar = 1$  [200, 201]:

$$\frac{d^2 r}{dt^2} = -2Q \text{grad } U(r) \cos^2 \left( -r \frac{dr}{dt} + \phi_0 \right), \quad (2.2.1)$$

where  $Q$  is the charge of particle,  $U(r)$  is the potential of external forces,  $\phi_0$  is the initial-phase. In one-dimensional case (motion along coordinate axis OX) the equation has the form:

$$\ddot{x} = -2QU'(x) \cos^2(-x\dot{x} + \phi_0) \quad (2.2.2)$$

That equation may be simplified if we put  $\dot{x} = v$  and consider  $v$ , i.e.  $\dot{x}$  as a function of  $x$ . The equation for  $v$  is following:

$$v \frac{dv}{dx} = -2QU'(x) \cos^2(-xv + \phi_0) \quad (2.2.3)$$

and it determines velocity  $\dot{x}$  as the function of  $x$  - so called phase curve. Sometimes that equation may be useful in investigations of the character of particle motion. The equation (2.2.2) does not possess the energy integral. But the following relation can be obtained with the help of (2.2.2):

$$\frac{1}{2} \dot{x}^2(t) - \frac{1}{2} \dot{x}^2(t_0) = -Q[U(x(t)) - U(x(t_0))] - Q \int_{t_0}^t \cos[2(-x\dot{x} + \phi_0)] dU \quad (2.2.4)$$

The energy integral for corresponding classical equation

$$\ddot{x} = -QU'(x)$$

may be expressed in form

$$\frac{1}{2}\dot{x}^2(t) - \frac{1}{2}\dot{x}^2(t_0) = -Q[U(x(t)) - U(x(t_0))] \quad (2.2.5)$$

The comparison of (2.2.4) with (2.2.5) allows obtaining some results about behaviour of velocity  $\dot{x}(t)$  for solution of equation (2.2.2).

In the case of two-dimensional motion the equations in coordinates system OXY are following:

$$\begin{aligned} \ddot{x} &= -2QU'_x(x, y)\cos^2(-x\dot{x} - y\dot{y} + \phi_0), \\ \ddot{y} &= -2QU'_y(x, y)\cos^2(-x\dot{x} - y\dot{y} + \phi_0). \end{aligned} \quad (2.2.6)$$

In three-dimensional case (in coordinate system XYZ) the equation of motion is analogous but the third similar equation for z and term  $-z\dot{z}$  are added to the cosine angle.

Analysis of equations in the coordinate system XYZ shows that they possess the area integral. Therefore, the particle is moving in some plane. That is why it may be considered as two-dimensional case only, i.e. equations (2.2.6).

Equations (2.2.6) possess the area integral that can be written in the form:

$$x\dot{y} - y\dot{x} = Const. \quad (2.2.7)$$

If the potential  $U(x, y)$  and, consequently, the force F depend on distance r only, then equations (2.2.6) may be simplified with the help of (2.2.7) as it is usually done in celestial mechanics. Viz., we shall pass from rectangular coordinates to the polar coordinates. If radius vector is denoted by r and polar angle by s, then equation for inverse distance  $u = \frac{1}{r}$  becomes more simple:

$$\frac{d^2u}{ds^2} + u = -\frac{2Q}{c^2u^2} F(u) \cos^2 \left( c \frac{du}{ds} \cdot \frac{1}{u} + \phi_0 \right), \quad (2.2.8)$$

where  $F(u)$  is expression of acting force as a function of a variable  $u$  and  $c$  is the constant of area integral.  $F(u) < 0$  corresponds to attractive force and  $F(u) > 0$  corresponds to repulsive force. The relation between inverse distance  $u$  and angle  $s$  is described by equation

$$\frac{ds}{dt} = cu^2. \quad (2.2.9)$$

Equation (2.2.8) for any force  $F(u) < 0$  has the stationary solution  $u = u_0 = Const.$ , that can be obtained from functional equation

$$u_0 = -\frac{2Q}{c^2u_0^2} F(u_0) \cos^2(\phi_0), \quad (2.2.10)$$

if that equation has positive solutions. Then each positive solution  $u_0 = u_0(\phi_0)$  corresponds to the stationary solution (circular orbit) depending on phase  $\phi_0$ . Hereinafter in section 2.9 you can see examples of such solutions.

The equations (2.2.2) and (2.2.6) possess particular solutions in the case of attractive force  $F(r)$  proportional to  $\frac{1}{r^3}$ . Viz.,

$$x(t) = a\sqrt{t}, \text{ and } \{ x(t) = a\sqrt{t}, y(t) = b\sqrt{t} \}, \quad (2.2.11)$$

where  $a, b$  are some constants that may be obtained from corresponding functional equations.

We obtain

$$\frac{a}{4} = 2Q \frac{\mu}{a^3} \cos^2\left(-\frac{a^2}{2} \phi_0\right)$$

in the case of Eq. (2.2.2) (if attractive force  $F(x) = -\frac{\mu}{x^3}$ ) and two analogous equations in the case of Eq. (2.2.6).

It may be noticed that equations (2.2.2), (2.2.6) do not possess such partial solutions in the case of analogous repulsive force.

We have not found any other particular solution of equation (2.2.2) and (2.2.6) or any integrals of these equations, so we have to use numerical integration for specific qualitative and quantitative analysis of motion characteristics.

### 2.3 Non - Autonomous Equations and Some Their Properties

*I just hardly can believe that usually nobody knows the details.*

*“Shaggy” thoughts. Stanislav Yezzy Lets.*

Particle motion equation in the field under some power influence in non-autonomous model of our theory has general view (2.1.1) [200, 201]. Being simplified it differs from autonomous equation (2.2.1) in term  $\frac{1}{2} \frac{dt}{dt} t$  to be added to cosine angle only. In one- and two-dimensional cases equations are formulated as follows:

$$\ddot{x} = -2QU'(x) \cos^2\left(\frac{1}{2} \dot{x}^2 t - x\dot{x} + \phi_0\right), \tag{2.3.1}$$

$$\ddot{x} = -2QU'_x(x, y) \cos^2 \left( \frac{1}{2} \dot{x}^2 t + \frac{1}{2} \dot{y}^2 t - x\dot{x} - y\dot{y} + \phi_0 \right), \quad (2.3.2)$$

$$\ddot{y} = -2QU'_y(x, y) \cos^2 \left( \frac{1}{2} \dot{x}^2 t + \frac{1}{2} \dot{y}^2 t - x\dot{x} - y\dot{y} + \phi_0 \right). \quad (2.3.3)$$

Motion equation in coordinate area XYZ has areas integrals like in autonomous case (2.2), so we can always consider motion in some field XY and use (2.3.2) equations. The latter also have area integral

$$x\dot{y} - y\dot{x} = Const.$$

If potential  $U(x, y)$  depends on distance  $r$  only, we may pass to polar coordinates  $(r, s)$ . Equation for  $u = \frac{1}{r}$  is the follows:

$$\frac{d^2 u}{ds^2} + u = -\frac{2Q}{c^2 u^2} F(u) \cos^2 \left( \frac{1}{2} c^2 \left( u^2 + \left( \frac{du}{ds} \right)^2 \right) t + \frac{c}{u} \frac{du}{ds} + \phi \right), \quad (2.3.4)$$

where  $F(u)$ - agent expression, as function  $u$ , and value  $c$  – constant of area integral. As far as time  $t$  belongs to the right part of equation, equation for  $t$  as  $s$  function should be added:

$$\frac{dt}{ds} = \frac{1}{cu^2} \quad (2.3.5)$$

Equations (2.3.4) and (2.3.5) represent system of two third -order equations in respect to variables  $u(s)$  and  $t(s)$  that can be numerically integrated. Equation (2.3.5) allows to correspond polar angle  $s$  changes lengthwise motion path with time  $t$ . Computations we made for some specific potentials underline that the character of autonomous and non-autonomous equation solutions some times are close in general, but in some cases they differ essentially. Hereafter you can see numerous examples of that fact (see sect.2.8).

Equations (2.3.1), (2.3.2) do not possess the energy integral. But it is possible

to compile the expressions analogues to (2.2.4) for autonomous equation and to derive some information about behavior of velocities  $\dot{x}(t), \sqrt{\dot{x}^2(t) + \dot{y}^2(t)}$  (see also sect.2.4).

It is of interest that considered equations have in the case of constant force field the particular solutions corresponding to the uniformly accelerated (decelerated) motion. Really, let be

$$U'(x) = -F_0 = \text{Const.}$$

and

$$U'_x(x, y) = -F_{0x} = \text{Const.}, \quad U'_y = -F_{0y} = \text{Const.},$$

Where  $F_0$  is acting force in the case of equation (2.3.1) and  $F_{0x}, F_{0y}$  are components of acting force in the case of equations (2.3.2). Then the function

$$x = at^2, \tag{2.3.6}$$

where  $a = 2QF_0 \cos^2(\varphi_0)$ , and the functions

$$x = at^2, y = bt^2, \tag{2.3.7}$$

where

$$a = 2QF_{0x} \cos^2(\phi_0), \quad b = 2QF_{0y} \cos^2(\phi_0),$$

satisfy our equations. If  $F_0 < 0$  (attractive force), then  $a < 0, b < 0$  and if  $F_0 > 0$  then  $a > 0, b > 0$ . Solution (2.3.7) describes a linear motion.

Also, equations (2.3.1), (2.3.2) have particular solutions in the case of external attractive force being proportional to  $\frac{1}{r^3}$ . Really, then

$$F(x) = -U'(x) = -\frac{\mu}{x^3} \quad (\text{in (2.3.1)})$$

and

$$F_{0x} = -U'_x(x, y) = -\frac{\mu x}{r^4}, F_{0y} = -U'_y(x, y) = -\frac{\mu y}{r^4}, \quad (\text{in (2.3.2)})$$

and mentioned particular solutions are following:

$$1) \quad x = a\sqrt{t} \quad (2.3.8)$$

where a is a real root ( $>0$  or  $<0$ ) of finite equation

$$a^2 = \sqrt{8Q\mu} \left| \cos\left(-\frac{3a^2}{8} + \varphi_0\right) \right|, \quad (2.3.9)$$

$$2) \quad x = a\sqrt{t}, y = b\sqrt{t} \quad (2.3.10)$$

where

$$a^2 + b^2 = d^2, d^2 = \sqrt{8Q\mu} \left| \cos\left(-\frac{3d^2}{8} + \varphi_0\right) \right| \quad (2.3.11)$$

It is necessary to make also two remarks about the structure of cosine argument in the above non-autonomous case.

1) The influence of terms  $\frac{1}{2}t\dot{x}^2$  and  $x\dot{x}$  must be always opposite, so it is more correct to write

$$\frac{1}{2}t\dot{x}^2 - |x\dot{x}| + \varphi_0.$$

2) The quantity  $\varphi_0$  is the initial phase and consequently the value of cosine argument at initial moment  $t=0$  must be equal  $\varphi_0$ , i.e.

$$\frac{1}{2}t\dot{x}^2 - |x\dot{x}| + \varphi_0 \Big|_{t=0} = \varphi_0. \quad (2.3.12)$$

If  $x(0) \neq 0, \dot{x} \neq 0$ , then the additional parameter is to be introduced and we must use instead (2.3.12) the following expression

$$\frac{1}{2}(t + t_*)\dot{x}^2 - |x\dot{x}| = \varphi_0, \quad (2.3.13)$$

where  $t_*$  is additional parameter (some global time?) satisfying the relation

$$\frac{1}{2}t_*\dot{x}^2(0) - |x(0)\dot{x}(0)| = 0. \quad (2.3.14)$$

## 2.4 Connection Between Equations with Oscillating Charge and Equations of Classical Mechanics

*All this looks like when the tail wags the dog.*

*A. Einstein.*

The equations of motion for a particle with oscillating charge have general form (2.1.1) or (2.1.14) [200, 201]. Assuming  $m = 1, \hbar = 1$  we obtain following equation in vector form:

$$\frac{d^2r}{dt^2} = -2Q\text{grad}U(r)\cos^2(\varphi), \quad (2.4.1)$$

where  $Q$  is the main part of particles' charge,  $U(r)$  is the potential of external forces and

$$\phi = \frac{t}{2} \left( \frac{dr}{dt} \right)^2 - r \frac{dr}{dt} + \phi_0 \quad (\text{non-autonomous-model}) \quad (2.4.2)$$

or

$$\phi = -r \frac{dr}{dt} + \phi_0 \quad (\text{autonomous model}) \quad (2.4.3)$$

Let consider the simple case of motion along x-axis and put  $Q=1$ . The corresponding equations are as follows:

$$\ddot{x} = 2F(x) \cos^2 \left( \frac{t}{2} \dot{x}^2 - x\dot{x} + \phi_0 \right) \quad (2.4.4)$$

or

$$\ddot{x} = 2F(x) \cos^2 (-x\dot{x} + \phi_0), \quad (2.4.5)$$

where  $F(x)$  is an external force ( $F(x)<0$  if attractive,  $F(x)>0$  if repulsive). The corresponding equation of classical mechanics is following:

$$\ddot{x}^0 = F(x^0). \quad (2.4.6)$$

It is of interest to compare solutions of these equations if  $F(x) \equiv F_0 = \text{Const.}$  (Constant field of force; for example, the field of plane condenser with the constant tension). Classical equation (2.4.6) describe in that case uniformly accelerated (decelerated) motion:

$$x^0(t) = \frac{1}{2} a t^2, \quad (2.4.7)$$

where  $a = F_0$  (under zero initial values). We do not have the general solution (in analytical form) of equation (2.4.4) even if  $F(x) \equiv F_0$ . Such solution has, apparently, sufficiently complicated form. But this equation possesses partial

solutions

$$x(t) = \frac{1}{2}qt^2, \tag{2.4.8}$$

where

$$q = 2F_0 \cos^2(\phi_0). \tag{2.4.9}$$

Such solution describes uniformly accelerated motion also, but the acceleration depends on value of initial phase  $\phi_0$  and varies from zero to  $2F_0$  (redoubled classical acceleration  $F_0$ ). If we observe the ensemble of particles with different initial phases  $\phi_0$  distributed uniformly on interval  $(0, \pi)$ , their averaged over ensemble acceleration is, apparently, near to classical. Besides, every classical solution (2.4.7) has its, so to say, the twin-solution (2.4.8), of equation (2.4.4) with correspondingly matched initial phase  $\phi_0$ .

It may be noticed that autonomous equation (2.4.5) does not possess partial solutions of form (2.4.8).

Non-autonomous equation has also the partial solution if attractive force  $F(x)$  is proportional to  $\frac{1}{x^3}$ . If  $F(x) = -\frac{\mu}{x^3}$ , then (see sect.2.3) this particular solution is following:

$$x(t) = a\sqrt{t}, \tag{2.4.10}$$

where  $a$  is a real root of finite equation

$$a^2 = \sqrt{8\mu} \left| \cos\left(-\frac{3a^2}{8} + \phi_0\right) \right| \tag{2.4.11}$$

The same particular solution of form (2.4.10) has the autonomous equation (2.4.5) (See sect.2.2) and also classical equation

$$\ddot{x} = -\frac{\mu}{x^3}. \tag{2.4.12}$$

In classical case  $a = \sqrt[4]{\mu}$ .

As was mentioned above, equations of form (2.4.4), (2.4.5) do not possess the energy integral, but is valid relation of form (2.2.4). Hence, if we write the classical energy integral in form (2.2.5), then it is possible to affirm following: velocity  $v^2(t)$  determined by equations (2.4.4), (2.4.5) differ from square of velocity  $v_0^2(t)$  at the same moment in solution of classical equation by following quantity:

$$\int_{t_0}^t F(x(t)\cos(\phi))dt, \tag{2.4.13}$$

where  $\phi = \phi(t)$  is equal to cosine argument in (2.2.4) or (2.2.5) correspondingly. Hence, value of  $v(t)$  may be more, than value of  $v_0(t)$  at the same moment and may be less or equal. It remains unclear, what will be the averaged velocity  $\tilde{v}(t)$  of ensemble of particles moving in accordance to equations (2.4.4), (2.4.5) with different initial phases uniformly distributed on segment  $(0, \pi)$ . Will be  $\tilde{v}(t) > v_0(t)$  or  $\tilde{v}(t) < v_0(t)$ , or  $\tilde{v}(t) = v_0(t)$  ?

Apparently, may be possible different variants in different scheme of motion and neither of these variants is not to be excluded.

Let us to determine some relations between classical equation and equations

with oscillating charge of our theory in the case of harmonic oscillator and to consider only the autonomous model. The analysis of non-autonomous model leads to analogous results, but all formulas are quite more complicated.

The standard equation of harmonic oscillations with frequency  $\omega$  is following:

$$\ddot{x} + \omega^2 x = 0 \tag{2.4.14}$$

Its solution

$$x(t) = a \cos(\omega t) \tag{2.4.15}$$

represents standard harmonic oscillation with zero initial phase. Let us consider the autonomous equation (2.4.5), i.e.

$$\ddot{x} = 2F(x) \cos^2(-x\dot{x} + \phi_0) \tag{2.4.16}$$

Certainly, function (2.4.15) does not satisfy this equation. We will set the inverse problem: what will be the force  $F(x)$  in (2.4.16) (and corresponding potential  $U(x)$ ) for its solution will be identical to (2.4.14)? For this purpose, we substitute (2.4.15) into (2.4.16) and obtain following expression for  $F(x)$ :

$$F(x) = \frac{-a\omega^2 \cos(\omega t)}{2 \cos^2\left(\frac{a^2 \omega \sin(2\omega t)}{2} + \phi_0\right)} \tag{2.4.17}$$

Using (2.4.15) we obtain the following expression for  $F(x)$ ;

$$F(x) = \frac{-\omega^2 x}{2 \cos^2(-\text{signum}(\dot{x})x\omega\sqrt{a^2 - x^2} + \phi_0)} \tag{2.4.18}$$

Hence, if the forces field  $F(x)$  is represented by (2.4.18), then equation (2.4.16) possess the solution  $x(t) = a \cos(\omega t)$ . It is possible to expand (2.4.18) in powers series in  $x$  provided  $|x| < a$ . In the simplest case  $\phi_0 = 0$  we obtain up to  $x^5$

following development:

$$F(x) = -\frac{1}{2}\omega^2 x - \frac{\omega^4 a^2}{2}x^3 - \frac{\omega^4}{6}(2\omega^2 a^4 - 3)x^5 - \dots \quad (2.4.19)$$

Corresponding expression for potential  $U(x)$  we obtain after integrating  $-F(x)$  in respect to  $x$ :

$$U(x) = \frac{1}{4}\omega^2 x^2 + \frac{1}{8}\omega^4 a^2 x^4 + \frac{1}{36}\omega^4(2\omega^2 a^4 - 3)x^6 + \dots \quad (2.4.20)$$

Certainly, these series converge sufficiently slowly for values of  $|x|$  near to  $a$ .

Let us to note that classical equation

$$\ddot{x} = F(x), \quad (2.4.21)$$

where  $F(x)$  is expressed by (2.4.18), does not possess the solution  $x(t) = a\cos(\omega t)$ . This equation possess the solution near to  $x(t) = a\cos(\frac{\omega}{\sqrt{2}}t)$ .

But if we choose corresponding value of initial phase  $\varphi_0$ , then equation (2.4.21)

will possess a solution near to  $x(t) = a\cos(\omega t)$ . Viz., if we set  $\varphi_0 = \frac{\pi}{4}$ , then up to  $x^3$

$$F(x) = -\omega^2 x + 2\omega^3 \text{asignum}(\dot{x})x^2 - 4\omega^4 a^2 x^3. \quad (2.4.22)$$

The equation (2.4.21), where  $F(x)$  is expressed by last formula, possess a solution near to  $x(t) = a\cos(\omega t)$  provided  $a$  is sufficiently small. It may be said in this case that theoretical motion of particle in some small neighbourhood of  $x = 0$  is nearly the same either described by classical equation (2.4.21) or by our equation (2.4.16). In particular, such motion near  $x=0$  described by our equation could satisfy approximately the integral of energy.

Certainly, if the value of initial phase  $\varphi_0$  is far from  $\frac{\pi}{4}$ , then particles' motion corresponding to classical and to our model may be quite different. It is natural because classical mechanics has suffered, as is well known, a failure by describing processes of micro-world, and we hope our model being more adequate.

## 2.5 Passage of Potential Step

*The language of truth is simple.*

*Seneca, Lucius Annaeus*

Overcoming the potential step is one of the simplest problems of Quantum Mechanics, especially in the case of a right-angle step. The standard quantum theory affirms following: if the kinetic energy of a particle is less than the potential energy of the barrier, then this particle is always reflected. At the same, there is always within standard quantum mechanics the probability of detecting this particle at some distance on the other side of the barrier (i.e. located on the top of the step) and that probability decreases exponentially with distance tending to zero. In other words, there is always some probability that the particle dives at first deep into barrier and later returns.

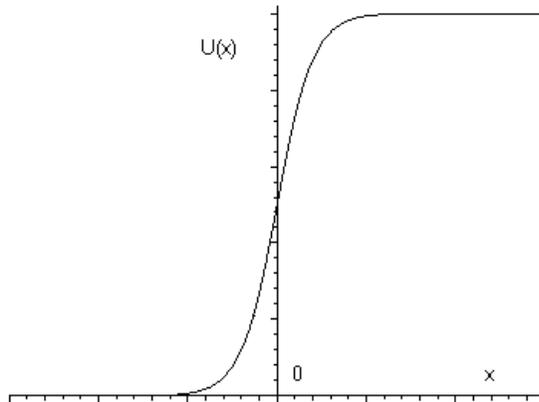
But the mentioned process is not well understood from the physical point of view. One may ask what causes the particle to return if it is located already on the horizontal top of the barrier. Nothing is affecting the particle; nothing prevents it from advance with constant speed. The reason and logic seem to be violated.

Our UGT removes such question. Let's consider the behavior of a particle using our theory and the equation with oscillating charge [172, 183, 200, 201].

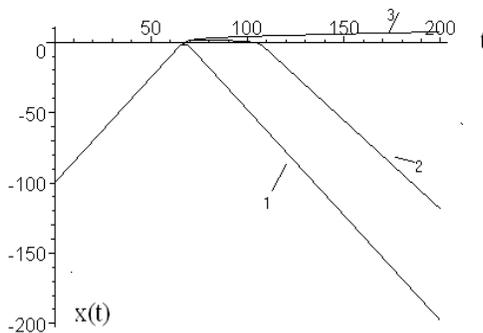
The numerical mathematical simulation of a right-angle potential is rather complicated. More over there is no such a real potential in the microcosm. So let us investigate that problem in more real case (Woods-Sacson modified potential):

$$U(x) = \frac{U_0}{1 + \exp\left(-\frac{x}{a}\right)},$$

where  $U_0 > 0$ ,  $a > 0$  (Fig. 2.5.1).



**Fig. 2.5.1** The step's potential  $U(x)$ .



**Fig. 2.5.2** Reflection (1 and 2) and passage (3) of particles for different values of initial phase.

The equation of motion (if  $a = 1$ ) is as follows:

$$m \frac{d^2x}{dt^2} + \frac{2U_0 \exp(-x)}{(1 + \exp(-x))^2} \cos^2 \left( m \frac{dx}{dt} x + \phi_0 \right) \quad (2.5.1)$$

in autonomous case or

$$m \frac{d^2x}{dt^2} + \frac{2U_0 \exp(-x)}{(1 + \exp(-x))^2} \cos^2 \left( \frac{m}{2} \left( \frac{dx}{dt} \right)^2 t - m \frac{dx}{dt} x + \phi_0 \right) \quad (2.5.2)$$

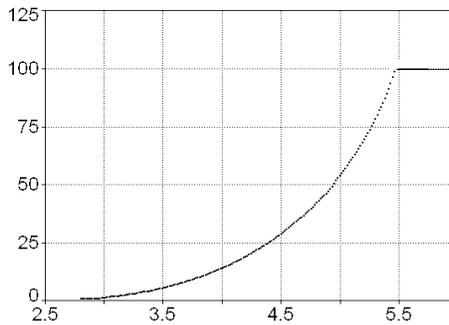
in non-autonomous case. The attempts to construct analytical solutions of these equations (including the cases of potentials like  $\arctan(x)$  or  $th(x)$ ) were not successful, and we used numerical integration for various initial data and initial phases. We calculated the trajectories ( $x$  as the function of time  $t$ ) for more, than 10 000 particles. Some trajectories are shown in Fig. 2.5.2. The trajectory 1 corresponds to straight reflection. The trajectory 2 can be explained as follows: the particle does not overcome the barrier but penetrates inside, some time moves within barrier, and later returns. The trajectory 3 shows that the particle penetrates into the barrier after the same interval of time as the particles 1 and 2, but thereafter moves away with very low speed and a vanishing charge. It is not the particle now but miserable remainders. From UQT wave packet point of view the particle is nearly absolutely spread throughout the cosmos, becomes a mathematical phantom.

We calculated also the number (the percentage) of all particles passing the barrier with respect to initial velocity (Fig. 2.5.3). The curve may be approximated well by an exponent. There was derived also the distribution curves (Fig. 2.5.4) for velocities and charges of passed particle. The calculations have revealed also the following features. Viz., at first, there is quite narrow

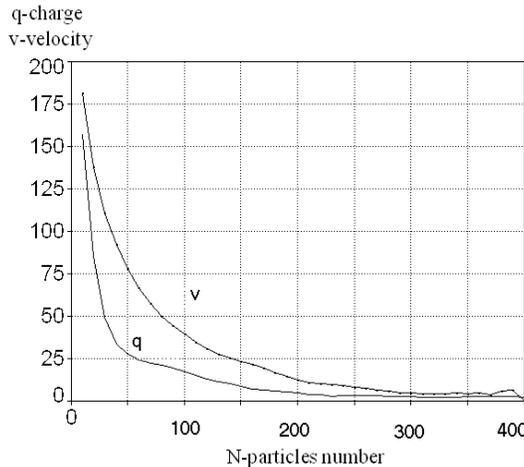
interval of initial phase  $\varphi_0$  values allowing a particle to penetrate a high barrier.

With the increase of barrier height that interval is narrowing around  $\frac{\pi}{2}$ . At

second, if the particle overcomes the barrier it comes away with very low speed and a vanishing small charge with the distance away and becomes a phantom (the example of the curve 3 in Fig. 2.5.2).



**Fig. 2.5.3** Number (percentage) of passed particles having uniform distributed initial phase as a function of their velocity.



**Fig. 2.5.4** Distribution of velocity  $v$  and charges  $q$  of passed particles. Number of particles is plotted on x-axis; quantities  $q$  and  $v$  are plotted on y-axis.

During our mathematical investigations we have not detected any fundamental difference in qualitative behavior in autonomous and non-autonomous models. Mathematical simulation of other potential barriers (namely, for  $\arctg(x)$  and  $\text{th}(x)$ ) resulted the same qualitative behavior.

We have obtained also the results of mathematical modeling of particle passing over potential barrier of following form:

$$U(x) = \frac{U_0 \left( \arctg(x) + \frac{\pi}{2} \right)}{\pi}$$

One-dimensional non-autonomous equation for the motion of the particle with mass  $m$  and with the constant part of charge  $Q$  is following:

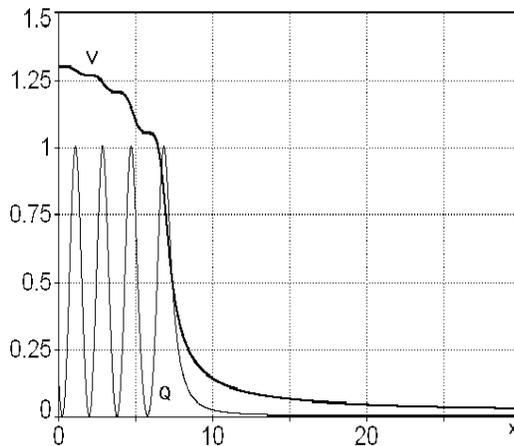
$$m \frac{d^2 x}{dt^2} + \frac{2QU_0}{(1+x^2)\pi} \cos^2 \left( \frac{mt}{2\hbar} \left( \frac{dx}{dt} \right)^2 - \frac{m}{\hbar} \frac{dx}{dt} x(t) + \varphi_0 \right) = 0$$

The plot of numerical solution of that equation for starting values:  $Q = \frac{\pi}{2}$ ,  $\phi_0 = 1.55$ ,  $x_0 = 10$ ,  $U_0 = 3/2$ ,  $\dot{x}_0 = 1.3$ ,  $\hbar = m = 1$  is shown in Fig. 2.5.5.

One can see typical horizontal steps at the left part of particles' velocity curve. There are seen the intervals, where the charge becomes vanishing small, no forces affect the particle, and it mechanically moves with nearly constant velocity. When the charge increases, the particle brakes and so on. That is why the oscillations of velocity can be seen. While approaching to the barrier the oscillating charge of particle abruptly decreases and the particle penetrates the barrier. Just after the barrier its velocity and its oscillating charge continue to decrease (exponentially) and further the particle may even disappear or becomes a phantom. In other words, according to our model, the particles do not turn back,

as usual quantum mechanics theory explains, but become a phantom and less detectable with moving away from the barrier.

However we did not detected the above-barrier reflection, well known within standard quantum mechanics. If the particle's energy is more than barrier potential then it always passes the barrier. Analytical evaluations for this problem in the cases of various steep curves of real steps (Heaviside's function is the limits case) will be very much appreciated.



*Fig. 2.5.5 Dependence of particle's velocity and its charge on distance from barrier.*

## 2.6 Tunneling Effects

*Nature is unsophisticated and does not tolerate the splendid magnificence of needless reasons.*

*Copernicus Nicolaus*

Any course of the quantum mechanics delves into details of that purely quantum effect. As we hope our audience is not restricted to specialists in quantum

mechanics we will undertake a short excursion into general classical mechanics [172, 183, 200, 201]. If there are two fields where particle potential energy is less within than at the interface dividing these areas, then the interface is called potential barrier. The most simple type of one-dimensional barrier is shown at Fig. 2.6.2, where datum line is potential energy in function of x axis coordinate. The point  $x_0$  potential energy is at its maximum  $U_0$  divides the whole interval  $(-\infty, \infty)$  in two domains,  $(-\infty, x_0)$  and  $(x_0, \infty)$ , where  $U < U_0$  always. Total energy of particle E equals the sum of it kinetic and potential energies

$$E = \frac{p^2}{2m} + U(x),$$

where m and p are the particle mass and impulse, respectively. Solving that equation with respect to momentum we get:

$$p = \pm \sqrt{2m(E - U(x))}$$

The sign must be chosen in accordance with the direction of particle motion. If  $p > 0$ , then the particle will approach the barrier from left to the right or if  $p < 0$  then in the opposite direction. Let us examine the particle moving from left to the right with total energy  $E < U_0$ . Then at some point  $x_1$  it potential energy will be  $U(x_1) = E$ , momentum will equal zero and, consequently, the particle will be stopped. The whole particle energy will be transformed in potential, and at pivot point  $x_1$  particle will start moving in opposite direction, unable to penetrate into the second area. Consequently, such potential energy fields (Fig. 2.6.1) are known as barriers as they prevent passage of a particle with  $E < U_0$ . The barrier is always transparent (or semi-transparent) in case  $E > U_0$ . For nuclear interaction, the specific term is Coulomb barrier the source of repulsive forces is

the charge of nucleus.

The Quantum Mechanics adds a new element to the picture. At  $E > U_0$  some particles may be reflected from the barrier and at  $E < U_0$  some particles can still pass the barrier. The effect is paradoxical for other reasons as well. If particle with  $E < U_0$  gets inside the barrier, it should have negative kinetic energy or imaginary momentum. However, that is the paradox of classical mechanics. Within quantum mechanics, the particle spends nearly no energy in overcoming the potential barrier; it seems to “tunnel” under the barrier. The details of exactly how the particle does this are unknown. The standard quantum mechanical “explanation” is that the particle’s behavior follows a wave probability and there exist the probability that the particle may be partially reflected and partially pass. That results in the appearance of probabilities of particle tunneling or reflecting. We are not going to show solutions of Schrodinger equation for tunnel effect you can find them in any course of quantum mechanics, we will only write the result for a barrier with the height  $U_0$  and width  $a$ . Probability of passing  $P$  is proportional to the following exponential function:

$$P \approx \exp\left(-\frac{2}{\hbar} a \sqrt{2m(U_0 - E)}\right) \quad (2.6.1)$$

Such approximate dependence remains for many types of barriers, although exact analytical solutions usually do not exist, but there are various opinions. One can see that contrary to the classical mechanics at  $E < U_0$  probability of barrier passage still exist. We should also note that in all events wave function amplitude in potential barrier area between points  $x=0$  and  $x=a$  is extremely small. Tunnel effect is significant when the power of the exponent in (2.6.1) close to unity.

$$\left(\frac{2}{\hbar} a \sqrt{2m(U_0 - E)}\right) \approx 1 \quad (2.6.2)$$

Suppose we have observed a particle  $E < U_0$  from inside the potential barrier, as particles penetrate it in accordance with (2.6.2). Then to detect the particle inside the barrier one should accurately fix its coordinates with the accuracy  $\Delta x < a$ . But in this case a mistake in calculating momentum is inevitable

$$\Delta p^2 > \frac{\hbar^2}{a^2}.$$

Replacing the value  $a$  from (2.6.2) will yield

$$\frac{\Delta p^2}{2m} > 2(U_0 - E)$$

In other words, measuring particle kinetic energy inside the barrier macro-device has an associate error that is twice the energy needed to escape the barrier. So, Nature preserves her building and tunneling secrets.

However, it is possible to make the situation clearer if using the equation with oscillating charge. When the particle approaches the barrier (particle energy is less than the barrier potential) while in a phase when its charge amplitude is very small, the barrier's repellent power is also small, and the particle is able to pass over such barrier. Fig. 2.6.1 illustrates the event. That phenomenon is unknown for standard quantum mechanics because according to it the phase of wave function does not play any essential role.

Examine passage of potential barrier in form of the Gaussian hat by the particle. Both autonomous and non-autonomous variants have been analyzed. One-dimensional potential and corresponding motion equations are the follows:

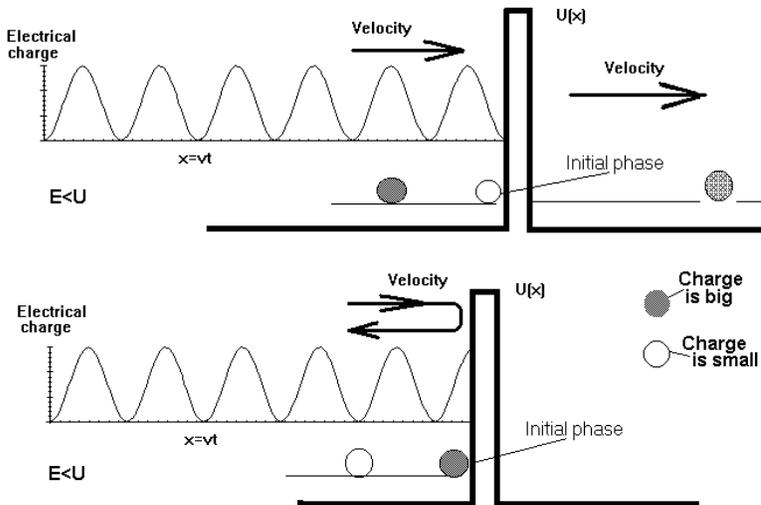
$$U(x) = U_0 \exp\left(-\frac{x^2}{\sigma^2}\right)$$

$$m \frac{d^2 x}{dt^2} + \frac{4U_0}{\sigma^2} Qx \exp\left(-\frac{x^2}{\sigma^2}\right) \cos^2(\phi) = 0,$$

where

$$\phi = \frac{mt}{2\hbar} \left(\frac{dx}{dt}\right)^2 - \frac{mx}{\hbar} \frac{dx}{dt} + \phi_0$$

for the non-autonomous case,  $\phi = -\frac{mx}{\hbar} \frac{dx}{dt} + \phi_0$  for the autonomous case.

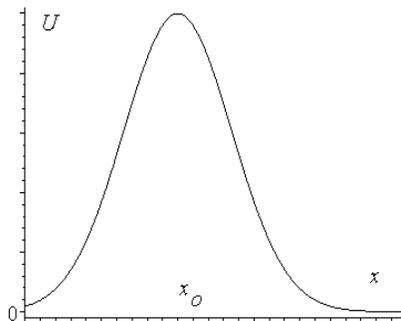


**Fig. 2.6.1** Visual picture of tunnel effect.

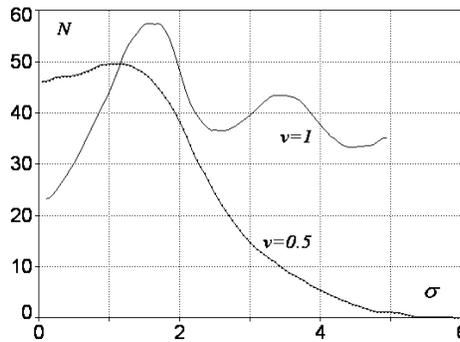
Both equations were solved numerically at  $m = Q = \hbar = 1$ ,  $U_0 = 0.5$ . The number of the particles passing the barrier was calculated (equivalent to the probability of barrier tunneling) depending on barrier width for randomly, uniformly distributed values of the initial phase within the interval  $\phi_0 = 0 \div \pi$  and fixed velocity. In Fig.

2.6.3 we can clearly see that the periodicity of tunneling probability depends on barrier width. Barrier back wall' reflection is an astonishing feature of nonlinear motion equations, because by intuitive form the particles' motion in monotone potential point of view the appearance of such effect is incomprehensible. It seems as though a nonlinear equation "remembers" what potential the particle have been moving against some time ago and "foreseen" what will be in future.

Then we considered the dependence of tunneled particle's number on its initial velocities and initial phases uniformly distributed the interval  $0 \div \pi$  for the same initial parameters. Plots in Fig. 2.6.4 are perfectly approximated by exponential functions of velocity corresponding with high precision to (2.6.1) or (2.6.4). That means that H. Heiger-J. Nuttall experimental law connecting the  $\alpha$ -disintegration constant with the velocity of emitted  $\alpha$ -particle disintegration may be theoretically derived from the evolved approach.



**Fig. 2.6.2** Potential barrier.



**Fig. 2.6.3** Dependence of number of particles passed barrier on barrier's width  $\sigma$  for different velocities.

Since the barrier transparency index is described by exponential function, it is possible to create a theory about the nature of  $\alpha$ -decay. According to it, when tunneling is of an extremely small probability ( $10^{-15}$  or less), that probability should sharply depend on the energy. Thus, let's change the particle's velocity approaching the barrier by a factor of four changes the probability of tunneling by 23 orders. We can now see that taking into account nuclear decay law [59] we will have an exponent with the other exponent as index that result in such strong dependence (H. Heiger-J. Nuttall law).

For a long time, the nature of alpha-decay was a mystery. Lord William Thomson Kelvin was the first to assume that particles emitted by radioisotope behave as if boiling inside "potential" crater. Statistically from time-to-time one of the particles receive enough energy to overcome the barrier, which is above the average energy of the particles inside. As it leaves, the particle is accelerated by potential field of the barrier, giving it even more energy. But E. Rutherford in his classical experiment disproved that view. During experiments uranium nuclei were bombarded by  $13 \cdot 10^{-6}$  erg alpha-particles from a thorium source. Alpha-particles propagation strongly depended on Coulomb law and according to

the Rutherford evaluations nuclear forces “came into play” at distances less than  $R_{\text{nucl}} = 3 \cdot 10^{-12}$  centimeter. It is clear that alpha particles are in the potential hole of uranium nucleus, which dimensions are at least less than  $R_{\text{nucl}}$ . But the uranium itself is radio-active and emits alpha particles with the energy  $6.6 \cdot 10^{-6}$  erg, so according to Kelvin’s model,  $13 \cdot 10^{-6}$  erg should be enough to overcome the Coulomb barrier and result in  $\alpha$ -capture by the uranium nuclei. Thus the experiment results in strange dilemma: either the Coulomb forces act differently upon incident and emitted alpha particles, or conservation of energy and momentum is entirely absent from these nuclear interactions. From our point of view that problem does not exist at all because the energy gained by the alpha-particle depends on its initial phase, as illustrated in Fig. 2.6.5.

Dependence on barrier width  $\sigma$  is not so simple. Let’s cite exact values of barrier transparency index  $D$  for the exact solution of the problem with rectangular barrier of width  $a$  to compare with general quantum mechanics results:

$$D = \frac{4E(E - U_0)}{4E(E - U_0) + U_0^2 \sin^2(ka)} \quad \text{if } E > U_0 \quad \text{and } k = \sqrt{E - U_0} \quad (2.6.3)$$

$$D = \frac{4E(U_0 - E)}{4E(U_0 - E) + U_0^2 \text{sh}^2(\gamma a)} \quad \text{if } E < U_0 \quad \text{and } \gamma = \sqrt{U_0 - E} \quad (2.6.4)$$

The expression (2.6.3) describes periodicity of the energy-tunneling index (sine function in denominator). That phenomenon is called over-barrier reflecting, but we have not found any over-barrier reflecting at  $E > U_0$  in the process of mathematical modeling. Vice versa, the expression (2.6.4) shows monotonous dependence of transparency index on the energy (hyperbolic sine function in denominator), at the same time our mathematical modeling shows oscillations

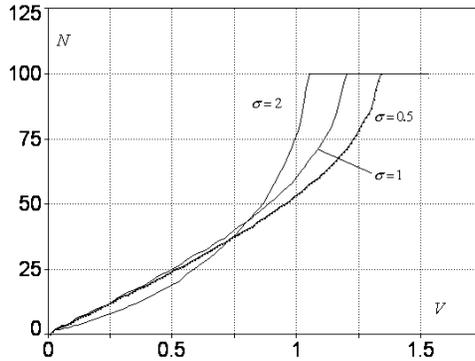
(see Fig. 2.6.3). That amazing result encourages, because from the Schroedinger wave equation point of view even now it is impossible to understand the reason of transparent index monotonous dependence on the barrier width  $a$  at  $E < U_0$ , when some periodicity is expected, and at the other side transparency index should become constant and equal to 1 at  $E > U_0$ , but it starts oscillating.

Later we analyzed the velocity of passed and reflected particles in comparison with velocity of incident particles. Input data for autonomous equation were the following:

$$x_0 = -10, v_0 = 0.8, U_0 = 20, \sigma = 0.5, m = 2, \hbar = 1.$$

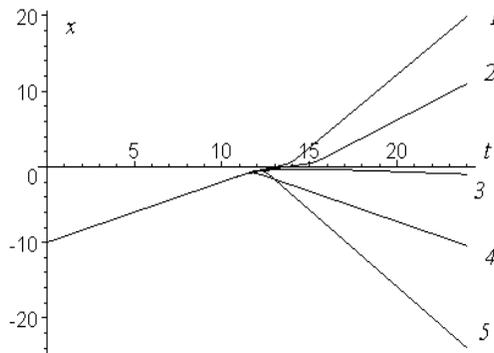
Initial phases were  $\phi_1 = 1.6, \phi_2 = 1.7, \phi_3 = 1.5, \phi_4 = 0, \phi_5 = 2.3$ . As it were expected, the particles velocities after passing or reflecting were smaller, equal or greater than those of incident particles. The results are shown in Fig. 2.6.5. Particle 1 passes and particle 5 is reflected with a higher velocity as they approached the barrier. Particle 2 passes and particle 4 is reflected with approximately the same velocity as they approached. Particles 3 are reflected by the barrier with a much smaller velocity.

That illustrate the fact that for single processes, described with the oscillating charge equation the conservation laws do not exist and they apparently appear after averaging over all initial phases. But the conservation law for the ensemble is rather complicated question, as far as the impulse sum before and after interactions do not equal each other but depend on potential. In the case of one or other potentials the value of impulse sum differs from each other and that question is still open.



**Fig. 2.6.4** Dependence of barrier transpance on velocity for different barrier width.

Amazingly, qualitative results in autonomous and non-autonomous cases appear to be similar



**Fig. 2.6.5** Coordinates of particle as function of time for different initial phases (1, 2-passing, 3, 4, 5-reflecting).

There exists an initial phase interval about  $\frac{\pi}{2}$ , where the high barrier is permeable even for particles with small energies. Here we have nearly the same problem as with the step barrier. Even the particles possessing very small energy and having the initial phase near  $\frac{\pi}{2}$  are able to pass (tunnel) the extremely high

barrier, but do it too long because they, so to say, snail inside the barrier. The charge is too small, the same is the acting on particle force and the motion with low velocity near to inertial motion may continue during very long period. It is quite natural to call that effect “snail”. That fact has been surely confirmed by numerous experiments (for example, [114]) and in different cases. Coulomb potential transparence exists (see sect.3.1), because in the other case it is impossible to explain the existing of pendulum orbits passing through the nucleus of Bohr-Sommerfeld atomic model (these orbits were simply excluded before from consideration as preposterous), and the atom s-states also.

Generally we have come from the analysis of the oscillating charge equations to some predicted physical conclusions.

Usually while solving the Shroedinger equation for the rectangular barrier it is assumed that only transmitted waves exist behind the barrier, because formally it does not have anything to be reflected from. But that idealization is not quite correct, because up to now whether the Universe is closed or not has been vague. And in the case of its insularity the reflected wave should exist. A rigorous mathematical approach requires the existence behind the barrier of transmitted and reflected waves that produce together spatial oscillation of the frequency function. Although absent from the general point of view, this requirement appears with our rigorous approach.

In that part conclusion we would like to state new version of Shroedinger equation solution (in the network of classical quantum mechanics) of the problem of charged particle passing through a potential barrier.

Assume that rectangular potential barrier has width  $a$  and height  $V_0$ . There is a particle with the energy  $E$ , less  $V_0$  and a mass moving towards the barrier.

Let us pick out on the axis OX three areas:

- I ( $x < 0$ )- before the barrier,
- II ( $0 < x < a$ )- inside the barrier,
- III ( $x > a$ )- behind the barrier. In each area Schroedinger equation is different.

Within the area I (particle does not reached the barrier jet) Schroedinger equation for the wave function, that we denote by  $\psi_1 = \psi_1(x)$ , is follows

$$\psi_1'' + k^2 \psi_1 = 0, \quad (2.6.5)$$

where

$$k^2 = \frac{2m_0 E}{\hbar^2}. \quad (2.6.5^*)$$

At some real initial value  $\psi_1(x_0), \psi_1'(x_0)$  in the point  $x_0 < 0$  that equation solution is usually written in the following view:

$$\psi_1(x) = A_1 e^{ikx} + B_1 e^{-ikx}, \quad (2.6.6)$$

where  $A_1, B_1$  – constants, expressed with the initial values, and terms  $A_1 \exp(ikx), B_1 \exp(-ikx)$  describe so-called direct and return waves.

The Schroedinger equation for the wave function  $\psi_2(x)$  within the area II has the following view

$$\psi_2'' - \chi^2 \psi_2 = 0, \quad (2.6.7)$$

where

$$\chi^2 = \frac{2m_0}{\hbar^2}(V_0 - E) > 0. \quad (2.6.7^*)$$

According to quantum mechanics literature the late equation solution analysis was done taking into account direct wave only in (2.6.6), return wave was excluded, considering reflected from the barrier and non-influencing onto the  $\psi_2(x)$  function. By the way, if we were trying to solve equation for  $\psi_2(x)$  from strongly mathematical base it would not be permitted to reject term  $B_1 \exp(-ikx)$ . The matter is that in terms of real initial values  $\psi_1(x_0), \psi_1'(x_0)$  function  $\psi_1(x)$  represents real harmonic oscillation. So coefficients  $A_1, B_1$  are complex conjugate. Assume that

$$A_1 = s + ir, B_1 = s - ir, \quad (2.6.8)$$

where  $S, r$  – real numbers. Then  $\psi_1(x)$  will be written in the following view

$$\psi_1(x) = 2M_1 \cos(kx + \phi), \quad (2.6.9)$$

where

$$M_1 = \sqrt{s^2 + r^2} = |A_1| = |B_1|, \quad (2.6.10)$$

$$\operatorname{tg}(\phi) = \frac{r}{s}, M_1 \cos(\phi) = s, M_1 \sin(\phi) = r,$$

Note that  $\phi$  phase may belong to any of trigonometric circle quarters. In particular, if  $\phi$  belongs to third quarter then  $S, r$  are negative.

General solution of the equation (2.6.7.) for  $\psi_2(x)$  in the area II is

$$\psi_2(x) = A_2 e^{-\chi x} + B_2 e^{\chi x} \quad (2.6.11)$$

However it is resulted from the physical point of view that rising exponent

$B_2 \exp(\chi x)$  will not be in the equation for  $\psi_2(x)$ . Wave function module is enable to rise inside the barrier. So the equation for  $\psi_2(x)$  in the view of:

$$\psi_2(x) = A_2 e^{-\chi x}. \tag{2.6.12}$$

By virtue of the process continuity at the border  $x = 0$  between the areas I and II the following conditions of functions  $\psi_1(x)$  and  $\psi_2(x)$  joining should be met:

$$\psi_1(0) = \psi_2(0), \psi'_1(0) = \psi'_2(0) \tag{2.6.13}$$

or

$$A_1 + B_1 = A_2, \quad ikA_1 - ikB_1 = -\chi A_2 \tag{2.6.14}$$

Evidently it is impossible to match that correlation if  $B_1 = 0$ . In the case  $B_1 \neq 0$ , further calculations are possible. In other words, previous correlation result in

$$A_2 = \frac{2ik}{ik - \chi} A_1 = \frac{2ik}{ik + \chi} B_1, \tag{2.6.15}$$

consequently

$$\frac{A_1}{B_1} = \frac{ik - \chi}{ik + \chi}, \tag{2.6.15*}$$

and taking into account (2.6.8), we have got the following correlation between  $s, r, k, \chi$ :

$$\frac{s^2 - r^2}{s^2 + r^2} = \frac{k^2 - \chi^2}{k^2 + \chi^2}, \quad \frac{sr}{s^2 + r^2} = \frac{k\chi}{k^2 + \chi^2}. \tag{2.6.16}$$

If we set

$$\frac{s}{r} = u, \frac{k}{\chi} = n, \quad (2.6.17)$$

and take into account (2.6.5\*), (2.6.7\*)

$$n = \sqrt{\frac{E}{V_0 - E}}, \quad (2.6.17^*)$$

then the result from (2.6.16) (leave out calculations) would be  $u = n$ . So values  $s, r$  should meet the requirement

$$\frac{s}{r} = \frac{k}{\chi} = \sqrt{\frac{E}{V_0 - E}} > 0. \quad (2.6.18)$$

The cases when both values  $s, r$  are more and less zero are permitted. Just while meeting the requirements of the last correlation for  $s, r$  we can find coefficient  $A_2$  in solution (2.6.12) for the function  $\psi_2(x)$ .

As far as  $|A_1|^2 = |B_1|^2 = s^2 + r^2$ , so from (2.6.18) follows that

$$r = \frac{1}{\sqrt{1+n^2}} |A_1|, \quad s = \frac{n}{\sqrt{1+n^2}} |A_1| \quad (2.6.19)$$

or

$$r = -\frac{1}{\sqrt{1+n^2}} |A_1|, \quad s = -\frac{n}{\sqrt{1+n^2}} |A_1|. \quad (2.6.19^*)$$

From (2.6.15) further follows (leaving out calculations), that

$$A_2 = \pm \frac{2n}{\sqrt{1+n^2}} |A_1| = \pm \frac{n}{\sqrt{1+n^2}} \max |\psi_1(x)|, \quad (2.6.20)$$

where the signs correspond to the signs of  $s, r$ .

Thus in order to make the solution  $\psi_2(x)$  in the area II in the form (2.6.12), the values  $s, r$  should meet relation (2.6.18) (according to (2.6.8), the coefficients  $A_1, B_1$  are to be expressed through these values).

Considering function  $\psi_1(x)$  as real harmonic oscillation expressed by formula (2.6.9), we obtain the same result. In particular, if  $\text{tg}(\varphi)$  meets the requirements of correlation (similar to (2.6.18))

$$\text{tg}(\phi) = \frac{\chi}{k} = \sqrt{\frac{V_0 - E}{E}}, \quad (2.6.21)$$

then we will get the following expression (similar to (2.6.20)) for  $A_2$ :

$$A_2 = \pm \frac{1}{\sqrt{1+\text{tg}^2(\phi)}} \max |\psi_1(x)| \quad (2.6.22)$$

If the requirement (2.6.18) (or equal condition (2.6.21)) was not fulfilled, then it would be impossible to find the coefficient  $A_2$  using the joining condition (2.6.14) of the solutions  $\psi_1(x), \psi_2(x)$ , i.e. solution  $\psi_2(x)$  in the form of (2.6.12) could not exist. That fact may be construed as follows: particle with wave function  $\psi_1(x)$  being within the area I cannot penetrate in that case into the barrier (interval II), in other words it cannot tunnel it. Note, that condition (2.6.21) of the barrier tunneling by the particle is the phase restriction for the harmonic wave accompanying particle motion.

So, let assume that the mentioned requirement has been fulfilled; the particle has penetrated inside the barrier with  $a$  width and is moving near internal barrier wall. Wave function  $\psi_2(x)$  expressed with the formula (2.6.12), is exponentially decaying in module with  $x$  growth from zero to  $a$ . If  $x = a$

$$\psi_2(a) = A_2 e^{-\chi a}, \quad \psi_2'(a) = -\chi e^{-\chi a} \quad (2.6.23)$$

and according to (2.6.20)

$$|\psi_2(a)| = \max |\psi_1(x)| \frac{n}{\sqrt{1+n^2}} e^{-\chi a}. \quad (2.6.24)$$

Let us now analyze function  $\psi_3(x)$  - Schroedinger equation solution in interval III ( $x > a$ ). Usually that equation for the function  $\psi_3(x)$  within interval III is written in the same form as equation (2.6.5) for function  $\psi_1(x)$  within area I:

$$\psi_3'' + k^2 \psi_3 = 0. \quad (2.6.25)$$

Note. Frankly speaking such equation does not correspond to the physical sense of the particle motion process within interval III. The fact is during particle tunneling the barrier it charge is decreasing. The decrease is more tangible the wider the barrier is. That corresponds to exponential decrease of wave function  $\psi_2(x)$  in module. So, it would be better to accept another value of particle energy for interval III,  $E_1 < E$  and consequently another value of frequency  $k_1 < k$ . However, it is unclear how this change should be determined. In any case, it is impossible to derive them from Schroedinger equation for intervals I and II. So we cannot exceed equation (2.6.25), but we should remember that this equation describes only an approximate model of the motion.

Let us write the general solution of the equation (2.6.25) in following form:

$$\psi_3(x) = A_3 e^{ik(x-a)} + B_3 e^{-ik(x-a)}, \quad (2.6.26)$$

where  $A_3, B_3$  - are coefficients to be determined so that functions  $\psi_2(x), \psi_3(x)$  meet at the boundary between interval II and III. These conditions are:

$$A_2 e^{-\chi a} = A_3 + B_3, \quad -\chi A_2 e^{-\chi a} = ikA_3 - ikB_3. \quad (2.6.27)$$

They coincide with correlation (2.6.14), if  $A_3, B_3$  interchanged with  $A_1, B_1$  and  $A_2 \exp(-\chi a)$  is exchanged for  $A_2$ . These substitution yield two equations in two unknown  $A_3, B_3$ , as far as  $A_2$  is already known. We will get the following expressions:

$$A_3 = \frac{1}{2} \left(1 + i \frac{\chi}{k}\right) A_2 e^{-\chi a}, \quad B_3 = \frac{1}{2} \left(1 - i \frac{\chi}{k}\right) A_2 e^{-\chi a} \quad (2.6.28)$$

or

$$A_3 = s_3 + ir_3, \quad B_3 = s_3 - ir_3, \quad (2.6.29)$$

where

$$s_3 = \frac{1}{2} A_2 e^{-\chi a}, \quad r_3 = \frac{1}{2} \frac{\chi}{k} A_2 e^{-\chi a}. \quad (2.6.29^*)$$

It is important that  $s_3, r_3$  meet the correlation:

$$\frac{s_3}{r_3} = \frac{k}{\chi} = n = \sqrt{\frac{E}{V_0 - E}}, \quad (2.6.30)$$

in other words – correlation coinciding with (2.6.18) for  $s_1, r_1$ . Then we get following formula for  $\max |\psi_3(x)|$

$$\max |\psi_3(x)| = 2\sqrt{s_3^2 + r_3^2} = |A_2| \sqrt{1 + \frac{1}{n^2}} e^{-\chi a}$$

or (taking into account formula (2.6.20) for  $A_2$ )

$$\max |\psi_3(x)| = \max |\psi_1(x)| e^{-\chi a} \tag{2.6.31}$$

Function  $\psi_3(x)$ , in the view (2.6.26) contains both direct and return waves and has the similar structure as function  $\psi_1(x)$ . We can also represent function  $\psi_3(x)$  as harmonic oscillation

$$\psi_3(x) = 2M_3 \cos[k(x - a) + \phi], \tag{2.6.32}$$

where

$$2M_3 = \max |\psi_1(x)| e^{-\chi a} \tag{2.6.32*}$$

Therefore, particle transmission index D equals

$$D = \frac{\max |\psi_3(x)|^2}{\max |\psi_1(x)|^2} = e^{-2\chi a} \tag{2.6.33}$$

Let us investigate the situation when on the way of the particle motion there is another, second, potential barrier with the same height  $V_0$  and width  $a_1$ , distanced from the first barrier at  $d_1$ . Then we should construct additional wave functions for the areas IV-inside the second barrier, V-behind the second barrier.

Function  $\psi_3(x)$  of the area III is expressed by formulas (2.6.26) or (2.6.32). At the second barrier border  $x = a + d_1$  that function equals

$$\psi_3(a + d_1) = A_3 e^{i r d_1} + B_3 e^{-i k d_1}, \tag{2.6.34}$$

where  $A_3, B_3$  are expressed by the formulas (2.6.28) – (2.6.29).

In the area IV equation for the function  $\psi_4(x)$  is to be written in the view similar to (2.6.7) with the same parameter  $\chi$ , i.e.

$$\psi_4'' - \chi^2 \psi = 0, \tag{2.6.35}$$

where

$$\chi^2 = \frac{2m_0}{\hbar^2} (V_0 - E).$$

Let us do the solution of that equation in the view similar to (2.6.12):

$$\psi_4(x) = A_4 e^{-\chi(x-a-d_1)}. \tag{2.6.36}$$

The relations joining conditions  $\psi_3(x), \psi_4(x)$  at the point  $x = a + d_1$  are similar to (2.6.14), i.e. are following:

$$A_3 e^{ikd_1} + B_3 e^{-ikd_1} = A_4, \quad ikA_3 e^{ikd_1} - ikB_3 e^{-ikd_1} = -\chi A_4. \tag{2.6.37}$$

Using the analysis made before in the case of (2.6.14), we may assert that conditions (2.6.37) are the same, in principle, and it is possible to determine the coefficient  $A_4$ , if  $A_3, B_3$  satisfy the following requirements. Viz., assume that

$$A_3 e^{ikd_1} = \tilde{s}_3 + i\tilde{r}_3, \quad B_3 e^{-ikd_1} = \tilde{s}_3 - i\tilde{r}_3, \tag{2.6.38}$$

then  $\tilde{s}_3, \tilde{r}_3$  should fulfill the following relations analogues to (2.6.18):

$$\frac{\tilde{s}_3}{\tilde{r}_3} = \frac{k}{\chi} = n = \sqrt{\frac{E}{V_0 - E}}. \tag{2.6.39}$$

Values  $\tilde{s}_3, \tilde{r}_3$  should be expressed according to (2.6.38) in  $s_3, r_3$  with next

formulas:

$$\tilde{s}_3 = s_3 \cos(kd_1) - r_3 \sin(kd_1), \tilde{r}_3 = s_3 \sin(kd_1) + r_3 \cos(kd_1). \quad (2.6.40)$$

As far as in accordance with (2.6.30)  $\frac{s_3}{r_3} = n$ , relations (2.6.39) will be fulfilled only in the case  $\sin(kd_1) = 0$ , i.e. if the distance  $d_1$  between the first and the second barriers is divisible by  $\frac{\pi}{k}$ :

$$d_1 = j \frac{\pi}{k}, \quad j=1,2,3,\dots, \quad (2.6.41)$$

where

$$k^2 = \frac{2m_0 E}{\hbar^2}.$$

In other case the particle will not be able to penetrate inside the barrier.

If condition (2.6.41) is met, then the conditions (2.6.37) are simplified:

$$A_3 + B_3 = A_4, \quad ikA_3 - ikB_3 = -\chi A_4 \quad (2.6.42)$$

or

$$A_3 + B_3 = -A_4, \quad ikA_3 - ikB_3 = \chi A_4, \quad (2.6.42^*)$$

These equations are totally congruent (second variant – to the sign  $A_4$ ) in structure with conditions (2.6.14). Values  $s_3, r_3, A_3, B_3$  expressed according to (2.6.29), meet relations (2.6.30), coincident with relation (2.6.18) for  $s, r$ . So, conditions (2.6.42) и (2.6.42\*) are compatible, now we derive as it were done above, the following formula for  $A_4$ :

$$A_4 = \frac{n}{\sqrt{1+n^2}} \max |\psi_3(x)| \quad \text{or} \quad A_4 = -\frac{n}{\sqrt{1+n^2}} \max |\psi_3(x)|. \quad (2.6.43)$$

Then while examining function  $\psi_5(x)$  within interval V and subordinating it to the equation

$$\ddot{\psi}_5 + k^2 \psi_5 = 0 \quad (2.6.44)$$

(with the same frequency  $k$  -- see above our note to the equation (2.6.25)), we obtain  $\psi_5(x)$  in form:

$$\psi_5(x) = A_5 e^{ik(x-a-d_1-a_1)} + B_5 e^{-ik(x-a-d_1-a_1)} \quad (2.6.45)$$

or

$$\psi_5(x) = 2M_5 \cos[k(x-a-d_1-a_1) + \varphi], \quad (2.6.46)$$

where the phase  $\varphi$  remains constant, and for  $A_5, B_5, M_5, \max |\psi_5(x)|$  we got the following formulas:

$$A_5 = \frac{1}{2} (1 + i \frac{\chi}{k}) A_4 e^{-\chi a_1}, \quad B_5 = \frac{1}{2} (1 - i \frac{\chi}{k}) A_4 e^{-\chi a_1}, \quad (2.6.47)$$

$$M_5 = |A_5| = |B_5|, \quad \max |\psi_5(x)| = \max |\psi_3(x)| e^{-\chi a_1}. \quad (2.6.48)$$

The barrier transmission index D equals

$$D = \frac{\max |\psi_5(x)|^2}{\max |\psi_1(x)|^2} = e^{-2\chi(a+a_1)} \quad (2.6.49)$$

Thus the probability of tunneling through two equal potential barriers depends on the particle's initial phase, energy and distance between barriers.

## 2.7 Passage of Potential Wells

*There are happen amazing adventures in the world...*

*A. S. Griboyedov*

In this section we will consider only one-dimensional problems. In classical mechanics the problem of rolling a particle into a finite-depth well is very simple from the physical point of view. Classical solutions of motion equations in the case of a potential well with symmetrical sides correspond to situation when a particle always rolls into the well and then leaves it at the same initial velocity. Moreover, in classical mechanics it is impossible to roll a particle into a well with symmetric sides in such a way that it remains there. It could be true but not for friction.

For the mechanics of a particle with an oscillating charge there are three possible modes of behavior, which, as it was found out, do not depend on the type of the potential well; it must only be finite and have equal sides:

1. A particle at small initial velocity and having certain initial phase can roll into the well and start oscillating there for a long time with damping, its charge will be constantly reduced, and finally this particle turns into “a phantom”. From our theory point of view, the wave packet representing this particle is spread all over the Universe. Moreover, there appears to be a certain threshold for the energy. If the energy is below this threshold, the particle will not roll out of the well at all. The value of the energy threshold depends on the type of potential. Oscillations without loss of energy and charge are also possible.
2. A particle can roll into the well and roll out at a speed higher, equal or lower than the initial speed. In other words, a particle passing the well can either

increase or reduce its energy. The energy conservation law for a single particle is not always valid. For details see Section 3.2.

3. A particle rolls into the well and starts oscillating there, and its energy will increase until it rolls out of the well with a much higher energy. It can even roll out in the direction opposite to the initial movement (reflection). Such processes seem to explain multiple experiments made by J. Griggs, Yu. Potapov, T. Misuno, A. Samgin, N. Tesla, R. Tandberg, P. Correa, etc. [60-69], but these phenomena will be discussed in Chapter 3.

The autonomous movement equation in the case of a potential well in the shape of hyperbolic secant [172, 183, 200, 201]

$$U(x) = -U_0 \operatorname{sech}(x^2) \tag{2.7.1}$$

will look like:

$$m \frac{d^2 x}{dt^2} + \frac{4U_0 Q x \cos^2 \left( mx \frac{dx}{dt} + \phi_0 \right) \sinh(x^2)}{\cosh^2(x^2)} = 0 \tag{2.7.2}$$

where  $t$ ,  $m$ ,  $Q$ ,  $\phi_0$  are mass, charge and initial phase of a particle respectively.

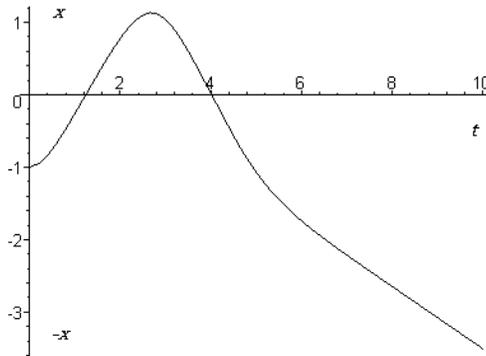
The plots below represent the results of a numerical solution of equation (2.7.2) by the Runge-Kutta-Merson method under following starting conditions:

$$U_0 = 1; m=1; Q=1; x_0 = -1; v_0 = 1/20.$$

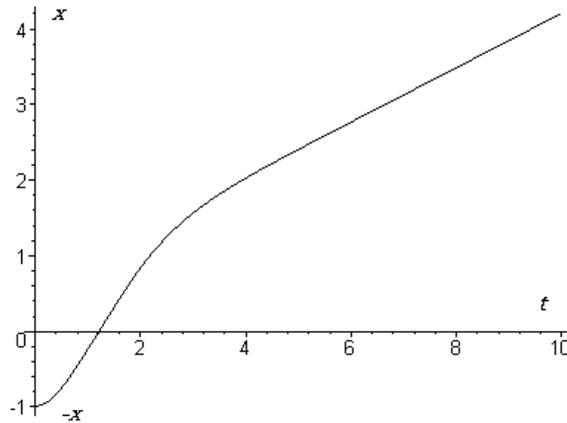
The resulting modes of the particle's behavior under equal starting conditions greatly depend on the initial phase, and its variations result in a very rich behavior. Let us demonstrate this fact. A particle with  $\phi_0 = 0.1$  is rolling into the well and back (is reflected) with a higher energy (Fig. 2.7.1). Under the same starting

conditions and with an initial phase value of  $\varphi_0=0.2$  passage of the particle through the well can be observed with nearly the same energy (Fig. 2.7.2.) and increasing oscillations inside the well is observing at  $\varphi_0=3.2$  (Fig. 2.7.3), where a particle can accumulate energy (a “Maternity Home” solution, for details see Section 2.8).

Certainly, such a process is not characteristic only in the case of the hyperbolic secant potential. Numerical researches of our problem with other potentials have been made, yielding similar results. (Remark: It was recently found out that hyperbolic secant potential plays a special role in quantum mechanics, and it turned out that barriers of this type are in general non-reflective [70], but for solutions of equation with an oscillating charge all this is not valid.)



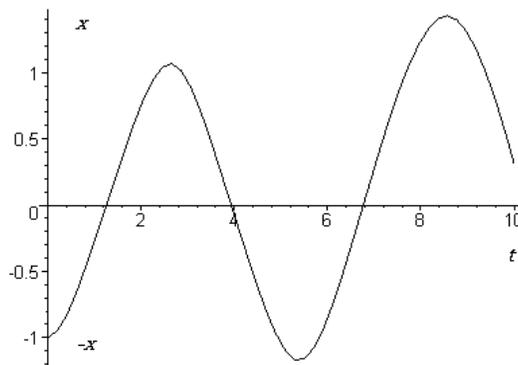
**Fig. 2.7.1** Reflection from potential well with a certain speed increase.



**Fig. 2.7.2** Passage of the well without reflection with a small energy change.

Let us take as an example a potential of the Gauss bell-curve:

$$U(x) = U_0 \exp\left(-\frac{x^2}{\sigma^2}\right) \tag{2.7.3}$$

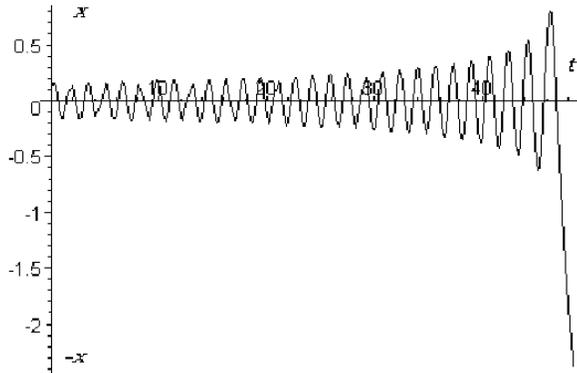


**Fig. 2.7.3** Oscillation in well with energy growth.

The movement equation is following:

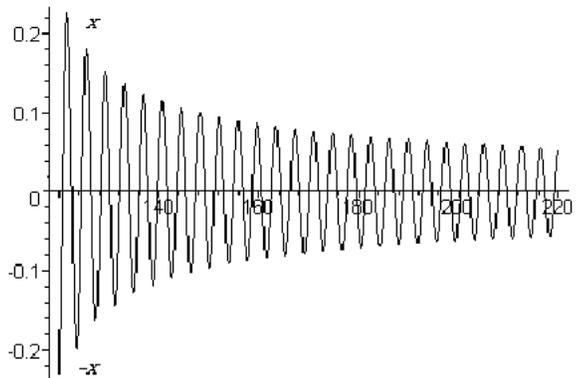
$$\frac{d^2x}{dt^2} + \frac{2Qx \exp\left(-\frac{x^2}{\sigma^2}\right) \cos^2\left(mx \frac{dx}{dt} + \phi_0\right)}{m\sigma^2} = 0$$

where  $m, Q, \phi_0$  are mass, charge and initial phase respectively. This equation has been solved under starting conditions  $Q=8, \sigma=1, m=1, x_0=0.1; v_0=0.5$  and for different initial phases. When initial phase  $\phi_0=0.1$  the particle oscillates, its energy increases and it overcomes the potential barrier, as can be seen in Fig. 2.7.4.



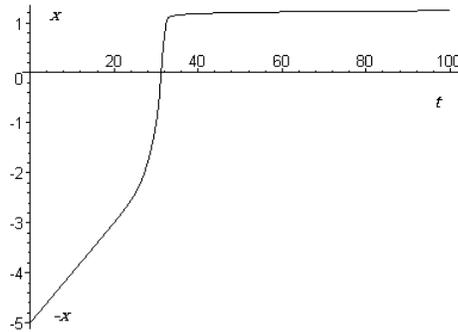
**Fig. 2.7.4** Passage of well with oscillations and energy growth.

For other starting conditions  $x_0=-4, v_0=0.01, \phi_0=2$  we get a process where the particle is spread all over the Universe, and not only its energy, but also its charge is reduced, and it turns into a “phantom”. Fig. 2.7.5 serves as an illustration.



**Fig. 2.7.5** Particle oscillation in well and its gradual disappearance.

Conditions under which the particle in the well does not disappear, but loses almost all its kinetic energy when leaving the well also exist. Thus, under the following starting conditions and parameters of the particle:  $m=5$ ,  $Q=10$ ,  $\sigma=1$ ,  $\phi_0=1.57$ ,  $x_0=-5$ ,  $v_0=0.1$  our simulation yields the result illustrated in Fig. 2.7.6.



**Fig. 2.7.6** Passage of well with nearly full loss of energy.

Such processes, however, are observed only in finite-depth wells, i.e. those having a bottom. In wells of the coulomb type or H. Yukawa type these processes do not take place, and no oscillations are observed (this does not mean, however, that they are totally absent there). The particle simply falls to the bottom of the well. In the Coulomb potential the fall happens approximately in accordance with the law:

$$x \sim at^{\frac{2}{3}}$$

Of course, the relativistic effect of mass accumulation will lead to the following relation in the limit:

$$x = ct .$$

If non-autonomous equations are used for modeling, the qualitative behavior

results will look the same, so we omitted them.

## 2.8 Harmonics Oscillator

*In fact, the chandelier in the Great Cathedral of Pisa was the first harmonic oscillator examined by Galileo Galley.*

*From newspapers*

Let us examine two variants of equations in the scalar case:

$$\ddot{x} = -2qx \cos^2(-x\dot{x} + \phi) \quad (2.8.1)$$

(autonomous equation) and

$$\ddot{x} = -2qx \cos^2\left(\frac{1}{2}\dot{x}^2 t - x\dot{x} + \phi\right) \quad (2.8.2)$$

(non-autonomous equation), where  $q$  is the constant part of particle's oscillating charge and  $\phi$  is the initial phase, that may be represented as  $\phi = \pi/2 + \varepsilon$ , where  $\varepsilon$  - phase deviation from  $\pi/2$ . As far as cosine is squared, it is quite enough to examine different values of  $\phi$  and  $\varepsilon$  within intervals from 0 to  $\pi$  or from  $-\pi/2$  to  $\pi/2$ .

The character of the particle motion to be described by these equations essentially depends just on  $\varepsilon$ . So we substitute equations (2.8.1), (2.8.2) for the following:

$$\ddot{x} = -2qx \sin^2(-x\dot{x} + \varepsilon), \quad (2.8.1^*)$$

$$\ddot{x} = -2qx \sin^2\left(\frac{1}{2}\dot{x}^2 t - x\dot{x} + \varepsilon\right). \quad (2.8.2^*)$$

The numerical integration of these equations yielded four types of solutions:

1. damped oscillations with amplitude, tending to zero; meanwhile particles sometimes assume a “phantom” state; in that case their wave packets are spread all over Universe;
2. irregular oscillations, remaining constant over a long period of time, thus yielding a quasi-stable situation;
3. oscillations with monotone increasing amplitude. In some cases these oscillations may abruptly enter a trajectory towards infinity; meanwhile cosine argument and the particle’s charge approach zero. It may be said that in that case the particle abruptly assumes a “phantom” state;
4. the particle almost immediately enters an escape trajectory and rapidly approaches the “phantom” state without any preliminary oscillations (it can be said without “preliminary doubts”).

In summary, only four variants of particle motion are possible: energy increase or decrease, stable and with vanishing particle (transformation into the “phantom” state).

Now we will consider only one-dimensional problems. In classical mechanics the problem of rolling a particle into a finite-depth well is very simple from the physical point of view. Classical solutions of motion equations in the case of a potential well with symmetrical sides correspond to situation when a particle always rolls into the well and then leaves it at the same initial velocity. Moreover, in classical mechanics it is impossible to roll a particle into a well with symmetric sides in such a way that it remains there. If not for friction this would be true.

There are allowed in the mechanics of a particle described by the equation with an oscillating charge solution with very different properties, i.e. allowed very different possible modes of particle’s behavior which greatly depend on the value

of initial phase in corresponding equations. There are very interesting from the standpoint of our UQT following modes of particle's behavior.

1. A particle can roll into the well and roll out (after certain period of oscillations or without oscillations) with higher (even much higher) velocity and energy than initial velocity and energy. We call the corresponding solutions as “Maternity home solutions” because the well takes in such case the part of “Maternity Home”, where are restored in essence to life new particles. The existence of such solutions seems to explain theoretically multiple experiments (Y. S. Potapov, 1993, 1998, A. Samgin, A. Baraboshkin et al., 1994, A. Samgin, 1995, T. Mizuno, M. Enio, T. Akimoto, K. Azumi, 1994, A. Patterson, 1996, C. Tinsley, 1995, J. Griggs, 1994, M. Huffman, 1995 and last Andrea Rossi.
  
2. A particle can roll out (after or without oscillations) or can remain to oscillate inside the well with much decreasing velocity and energy tending to zero. The corresponding solutions we call as “Crematorium solutions”. Such particles turn out into “phantom” and wave packets representing such particles are spread over the Universe.
  
3. A particle can also preserve stationary oscillations with constant amplitude of classical type inside the well.

The plots below (Fig. 2.8.1 - Fig. 2.8.7) illustrate these modes of particle's behavior. These plots have been obtained after numerical integration of the autonomous equation

$$m \frac{d^2 x}{dt^2} + \frac{4U_0 Qx \cos^2 \left( mx \frac{dx}{dt} + \phi_0 \right) \sinh(x^2)}{\cosh^2(x^2)} = 0 \quad (2.8.3)$$

in the case of the potential well in the shape of hyperbolic secant

$$U(x) = -U_0 \operatorname{sech}(x^2) \tag{2.8.4}$$

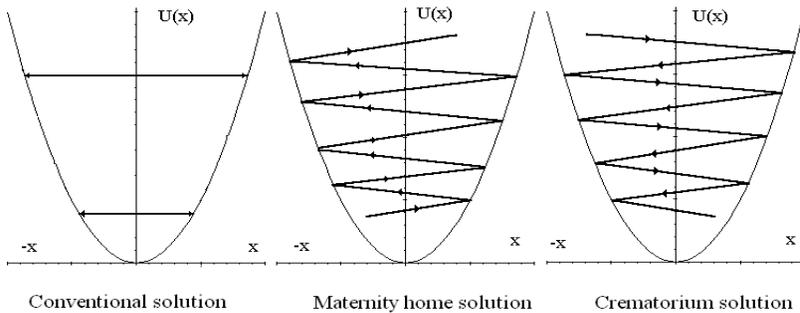
where  $m, Q, \varphi_0$  are mass, charge and initial phase of a particle respectively.

Numerical solutions in all six cases were obtained under following values of  $m, Q, \varphi_0$  and initial data:

$$m=1, Q=1, U_0 = 1, x_0 = -0.5, \dot{x}_0 = 1/20$$

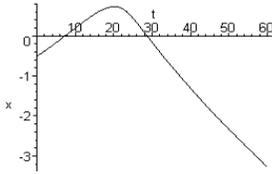
The trajectories on Fig. 2.8.2, Fig. 2.8.4, Fig. 2.8.5 represent the “Maternity Home” solutions. Velocity of particles after they rolled out the well are at  $t=0$ ,  $\dot{x}(0) \sim 0.9$ , i.e. almost 20 times greater than initial  $\dot{x} = 1/20$ .

The trajectory on Fig. 2.8.2 represents also the “Maternity Home” solution although the increase of velocity is not so essential:  $x(0) = 0.094$  at  $t=100$  only nearly two times more.

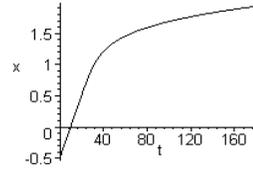


**Fig. 2.8.1** Possible solutions for the harmonic oscillator.

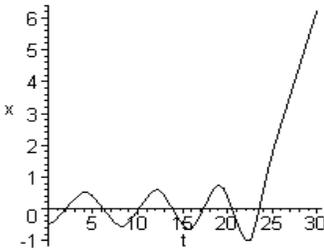
The trajectories on Fig. 2.8.3, Fig. 2.8.6 represent the “Crematorium” solutions. The first  $x(0)$  particle leaves the well and moves away with monotonously decreasing velocity and spread out over all Universe. The second particle is oscillating inside the well with slowly decreasing and tending to zero velocity.



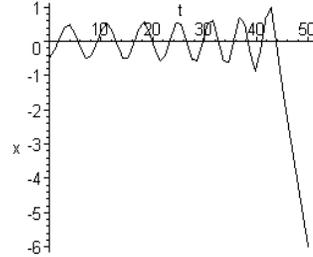
**Fig. 2.8.2** “Maternity home” solution.



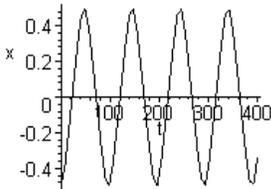
**Fig. 2.8.3** “Maternity home” solution.



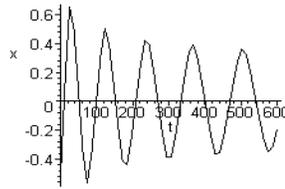
**Fig. 2.8.4** “Maternity home” solution.



**Fig. 2.8.5** “Maternity home” solution.



**Fig. 2.8.6** Classical-stable solution.



**Fig. 2.8.7** “Crematorium” solution.

These solutions have been reported for the first time by one of the authors at the conference ICCF5 taking place in Monte-Carlo [83] and published in [55-57, 82-86, 104, 123-125], and called: «Maternity Home», «Crematorium», stable and “Ghostly”. The first three solutions correspond, in general, to Fig. 2.8.1. The solution passing into “Phantom” or “Ghostly” state has analogous to solutions of Shroedinger’s equation containing Hermite functions, because the exponential “tails” of the wave function exist always out of parabolic well.

Such solutions become possible only because the Energy Conservation Law for the case under consideration does not work for such motion equations that

result in deep sequence, more detailed examination of which will be done in parts 3.2, 3.3 and 3.4.

## 2.9 Kepler Problem

*In 50es N. Bohr recollected with smile the event when after one of the lecturesthe student asked him: “Were there in reality such morons to think that electronrotating by orbit?”*

Let us examine the motion of the particle with oscillating charge (its constant part is equal to  $q$ ) around attractive (according to Coulomb law) fixed central nucleus with the charge (of opposite sign)  $q_0$  [172, 183, 200, 201]. The motion of such particle is flat due to area integral existence, so we may examine the motion equations on the plane OXY. For the autonomous model these equations are following:

$$\ddot{x} = -\frac{2\mu x}{r^3} \cos^2(-x\dot{x} - y\dot{y} + \phi), \ddot{y} = -\frac{2\mu y}{r^3} \cos^2(-x\dot{x} - y\dot{y} + \phi), \quad (2.9.1)$$

where phase  $\phi$  varies within the interval  $[0, \pi)$  and  $\mu$  depends on  $q$  and  $q_0$ .

For the particle motion process examining we consider expediential to change coordinates for polar  $r$  (radius-vector),  $s$  (azimuth) and handle the equation relative to polar inverse distance  $u = 1/r$ . The equation relative to  $u$  has the following view:

$$u'' + u = \frac{\mu}{c^2}, \quad (') = d / ds, \quad (2.9.2)$$

and  $u, u'$ , integral constants are connected with  $x, y, \dot{x}, \dot{y}$  by the following formulas of celestial mechanics:

$$x\dot{y} - y\dot{x} = c, x\ddot{x} + y\ddot{y} = -c \frac{u'}{u}, \dot{s} = cu^2. \quad (2.9.3)$$

Within our theory the equation with oscillating charge relative to  $u$  has the following view:

$$u'' + u = \frac{2\mu}{c^2} \cos^2 \left( c \frac{u'}{u} + \phi \right) \quad (2.9.4)$$

As it is known the equation (2.9.2) has stationary solutions  $u = \text{Const.}$ , corresponding to stationary motion in a circle orbit, as well as solutions  $u = u(t)$  corresponds to motion (also stationary) along elliptic orbits. Each of these motions is orbital stable. Small change of initial conditions corresponds, with rare exception, to small changes of the orbit value. The solutions of the equation (2.9.2) can be represented in simple analytical form. Our equation (2.9.4) describes much more complicated motions and representation of its solution in analytical form is scarcely possible. But it is quite interesting that this equation also has stationary solution  $u = \text{Const.}$  corresponding to motions in a circle orbit, namely,

$$u = u_0 = \frac{2\mu}{c^2} \cos^2 \varphi. \quad (2.9.5)$$

Each initial phase  $\varphi$  answers its own circle orbit. Besides, we are able to examine the stability of this solution with the help of so-called variation equation. If we set

$$\mu / c^2 = a$$

and

$$u = u_0 + w,$$

then the variation equation (in respect to  $w$ ) has the following view

$$w'' + a \frac{2c}{u_0} \sin(2\phi)w' + w = 0. \tag{2.9.6}$$

From that equation it is obvious that for  $\sin(2\phi) > 0$ , i.e. for  $0 < \phi < \pi/2$ . the solution  $u = u_0$  is stable, and for  $\sin(2\phi) < 0$ , i.e. for  $\pi/2 < \phi < \pi$  is unstable. For  $\phi = 0$  и  $\phi = \pi/2$  variation equation results in nothing (does not have any result).

But at  $\phi = \pi/2$  we have  $u_0 = 0$  (according to (2.9.5) and  $r_0 = \frac{1}{u_0} = \infty$ , i.e. for  $\phi \rightarrow \pi/2$  the circle orbit radius tends to infinity.

Such results are confirmed by the plots calculated with the help of numerical integration of equation (2.9.4). We have assumed that in (2.9.4)  $\mu = 0.5$  and at the initial moment  $t = 0$  the particle is placed on the right side of the center ( $s = 0$ ) at the distance  $r_0 = 1$  and had the velocity  $v_0 = 1$ , upward polar axis. If using coordinates  $x, y$ , then their initial values are follows:

$$x(0) = 1, y(0) = 0, \dot{x}(0) = 0, \dot{y}(0) = 1.$$

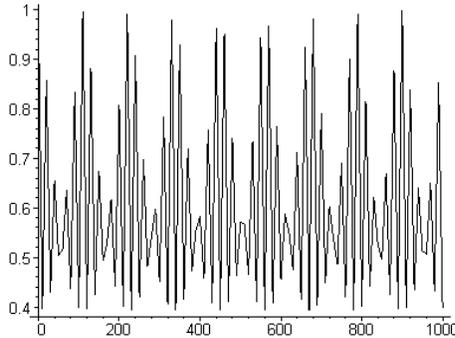
Using formulas (2.9.3), we will get the initial values for  $u, u'$  and the value of the area constant  $c$ :

$$u(0) = 1, u'(0) = 0, c = 1.$$

We have numerically integrated the equation (2.9.4) under that initial data and different values of the phase  $\phi$ . In Fig. 2.9.1, ..., 2.9.5 there are plots for  $r$  as  $s$  function at  $\phi = 0, 0.1, \pi/3, \pi/2, 1.8$ , constructed not within the polar coordinates but within rectangular one, where abscissa axis corresponds to  $s$ , and ordinate axis corresponds to  $r$ .

First plot (for  $\phi = 0$ ) shows that apparently stationary solution at terms that  $\phi$  is

stable. The next four plots replay to our conclusions about stability and instability of stationary solutions. Elliptical stationary solutions have not been found yet, however that does not mean they are not exist at all.

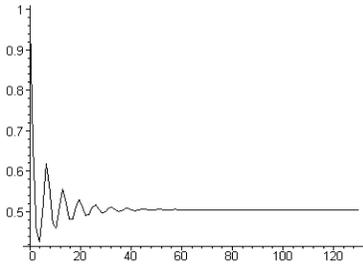


**Fig. 2.9.1**  $\varphi = 0$ .

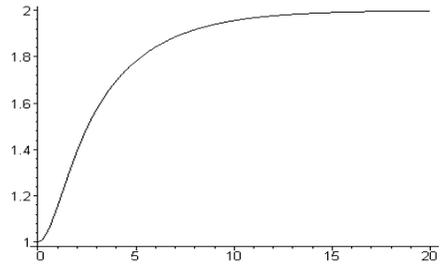
Within the non-autonomous model the motion equations differ from the equation (2.9.1) in terms  $\frac{1}{2}t(\dot{x}^2 + \dot{y}^2)$  added to argument  $-x\dot{x} - y\dot{y} + \varphi$  of cosine.

We have made a lot of numerical integration of these equations under numerous initial conditions and initial phases. In general the obtained solutions are close by its character to solutions of autonomous equations, but we were not lucky to find stationary solutions. The same problem has been studied in part 1.3 above.

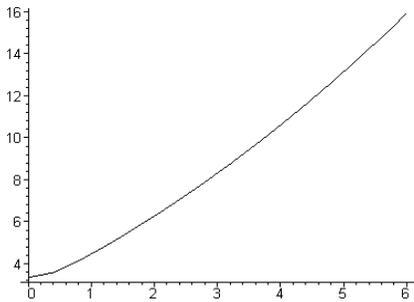
It is well know that Bohr-Sommerfeld model perfectly describes the hydrogen atom spectrum. In one very serious book regarding that problem (we are specially do not mention in which exactly) it is written that this fact is “a result of conflicting concept and spin absence compensating each other (!!!)”.



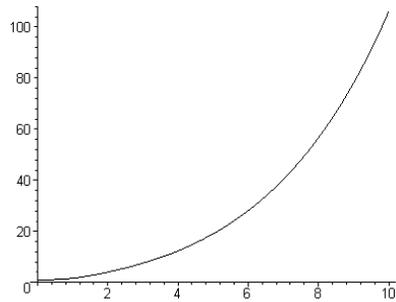
**Fig. 2.9.2**  $\phi = 0.1$



**Fig. 2.9.3**  $\phi = \pi/3$



**Fig. 2.9.4**  $\phi = \pi/2$



**Fig. 2.9.5**  $\phi = 1.8$

## 2.10 Kepler Problem (Scattering of Particle on Coulomb Potential)

*What were, that will be, what were done, that will be done; there is nothing new under the sun. There is something speaking about “look that is new”; but that was already in the ages before us. There is no memory of the former, and those who will be after us will not keep memory.*

*Ecclesiast, 9, 10, 11*

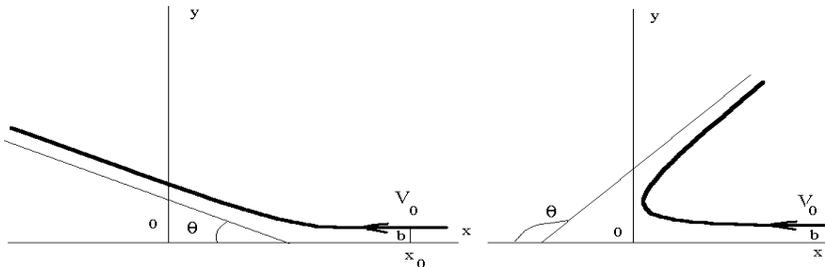
The study of the process of the particles scattering is the main method of the

elementary particles physics study. Rutherford was the first who applied such analysis and discovered the existence of heavy nucleus in the atoms. By using common Coulomb's law and Kepler equation he derived formula for the dependence of the scattering angle from the sighting distance and speed of a flying particle:

$$tg\left(\frac{\theta}{2}\right) = \frac{zZe^2}{mv^2b} \tag{2.10.1}$$

The same correlation was obtained in Quantum theory. Experiments made by Rutherford confirmed the validity of this scattering formula, and he was very proud of this because formula was derived on the basis of classical points without any quantum theory ideas.

It should be noted that Coulomb potential had been already known from the other independent experiments, and the scattering problem became leading in the process of interaction potential determining. But in spite of the numerous measuring of the hadrons scattering processes the potential of the strong interaction has not been rendered yet. This is not simply by coincidence, because the modern quantum theory couldn't compute either electron charge or elementary particles masses while it is possible with UQT [1-15].



**Fig. 2.10.1** Particle Path after Scattering.

Moreover no low-energy nuclear reactions are possible in standard quantum theory even they have been confirmed by experiments long ago [16]. The

phenomenon of chemical catalysis well explained in UQT [17] is also incomprehensible. The proper solution of the problem of a wave packet scattering at another wave packet is a too distant future due to nonlinear nature, today we cannot even imagine how we can come to grips with the strong settings of this problem. Below we are going to solve the classical problem of scattering of the particles at Coulomb and short-range potentials for the oscillating charge equation, that is by the authors opinion is more adequate.

First of all we should demonstrate that application of the equation with oscillating charge does not conflict with the formula (1). Equation with oscillating charge may be written in both autonomous and non-autonomous forms. The properties of such equations are discussed in details in [172, 183, 200, 201]. Further we are going to discuss the solution for autonomous equations as far as solution of the problem of scattering for the non-autonomous equation has similar results but more intricate. Autonomous equation with oscillating charge for arbitrary potential has the following form [195]:

$$m \frac{d^2 r}{dt^2} = 2Q \text{grad}U(r) \cos^2(-m \frac{dr}{dt} + \varphi) \quad (2.10.2)$$

$$\frac{d^2 x}{dt^2} = 2Q \frac{x}{r^3} \cos^2(-x \frac{dx}{dt} - y \frac{dy}{dt} + \varphi);$$

$$\frac{d^2 y}{dt^2} = 2Q \frac{y}{r^3} \cos^2(-x \frac{dx}{dt} - y \frac{dy}{dt} + \varphi) \quad (2.10.3)$$

where  $r = \sqrt{x^2 + y^2}$ .

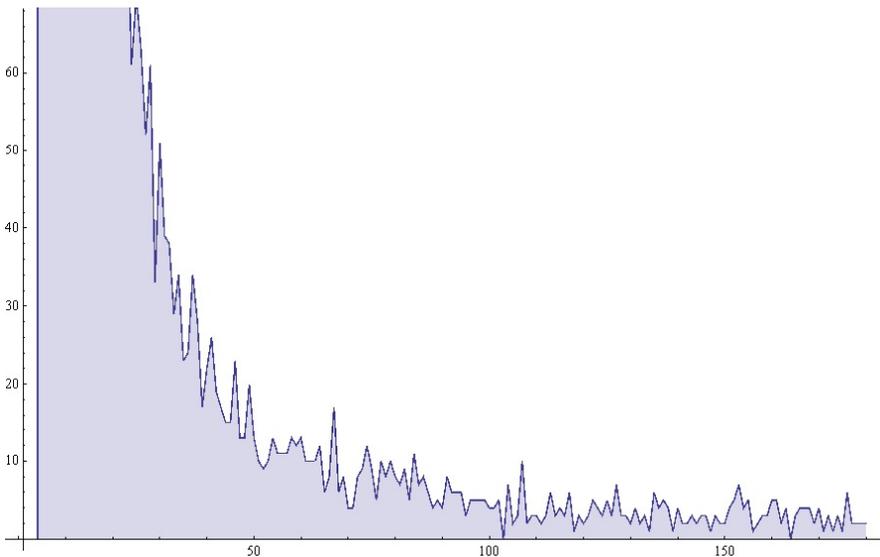
The same equations can we written for the classical charge:

$$\frac{d^2 x}{dt^2} = Q \frac{x}{r^3}; \quad \frac{d^2 y}{dt^2} = Q \frac{y}{r^3} \quad (2.10.4)$$

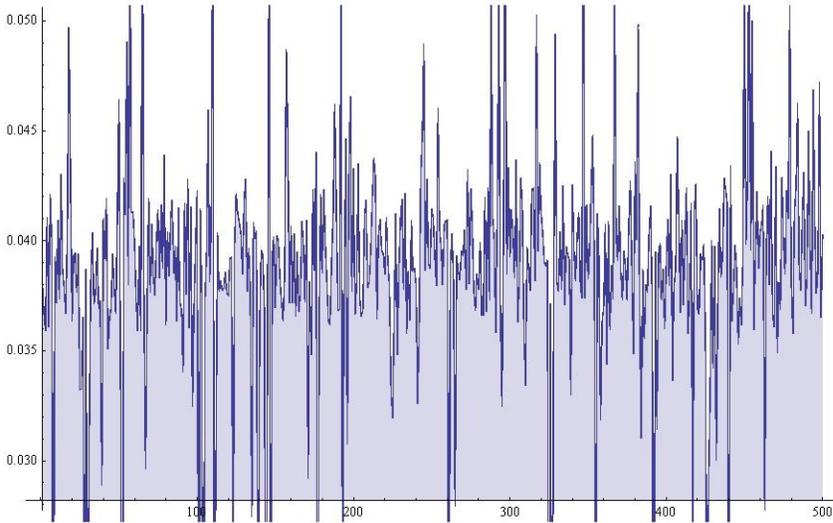
The systems 2.10.3 and 2.10.4 were numerically solved for similar initial terms:

$$Vx_0 = 5, x_0 = -1000, b = 0 \div 1, \varphi = 0 \div \pi, Vy_0 = 0, \Delta T = 1000,$$

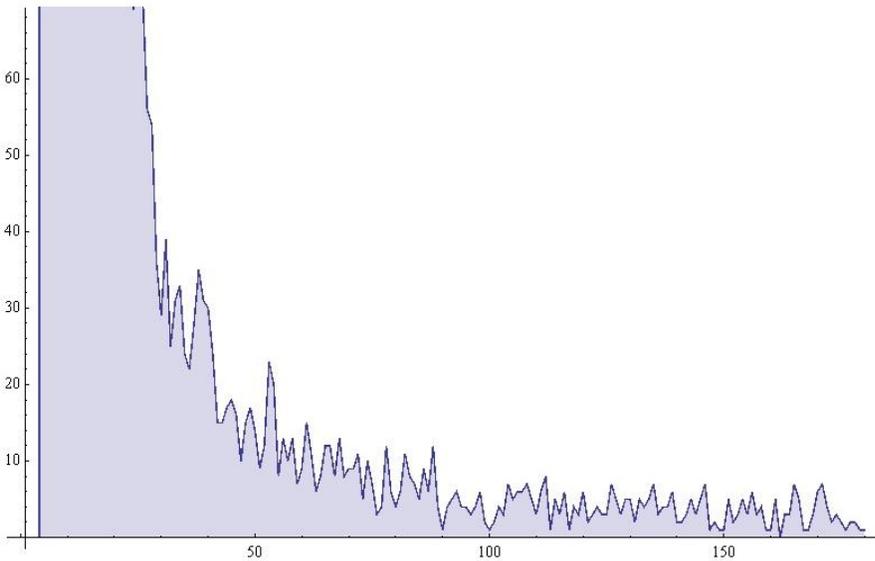
number of particles  $N=10000$ . Each calculation is made for randomly chosen initial phase and sighting distance. The Fig. 2.10.2 and Fig. 2.10.4 show the dependence of the number of scattered particles on the scattering angle and they are practically coincident. The Fig. 2.10.3 and Fig. 2.10.5 demonstrate the dependence of  $btg \frac{\theta}{2}$  for 500 randomly chosen particles; it should be constant for the solution of equations 2.10.3 and 2.10.4. As we can see from the diagrams this condition is fulfilled. But it's quite amazing as for the equation 2.10.4 the deviation angle  $\theta$  depends on the sighting distance and the energy only, while for the solution of the equation 2.10.3 the angle  $\theta$  depends on the initial phase also. The coincidence appears because the Coulomb potential varies very slowly. Thus the scattering at the Coulomb potential is correctly described by the equation with the oscillating charge.



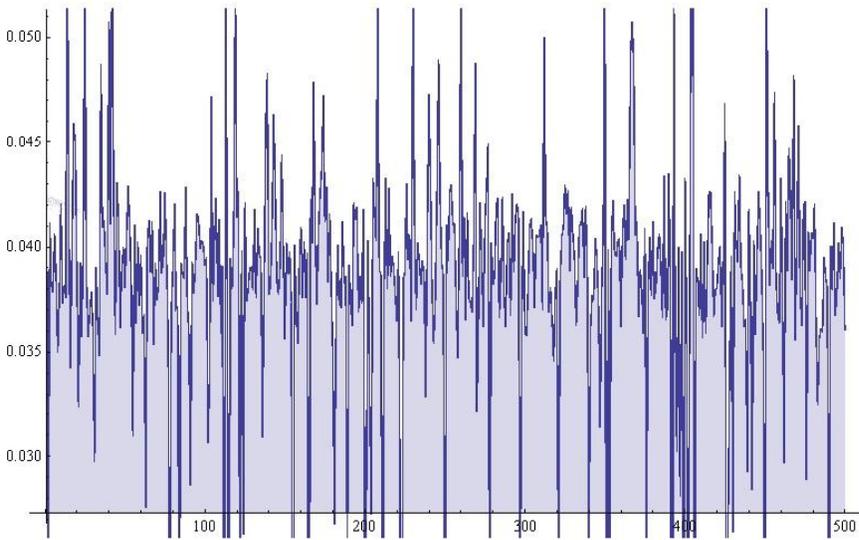
**Fig. 2.10.2** Dependence of Scattered Particles Number from the Angle for the Equations (2.10.3).



**Fig. 2.10.3** Value  $btg\left(\frac{\theta}{2}\right) \sim \frac{1}{v^2}$  for the Equation (2.10.3) for 500 Random Particles.



**Fig. 2.10.4** Dependence of the Scattered Particles Number from the Angle for the Equations (2.10.4).



**Fig. 2.10.5** Value  $\text{btg} \left( \frac{\theta}{2} \right) \sim \frac{1}{v^2}$  for the Equation (2.10.4) for 500 Random Particles.

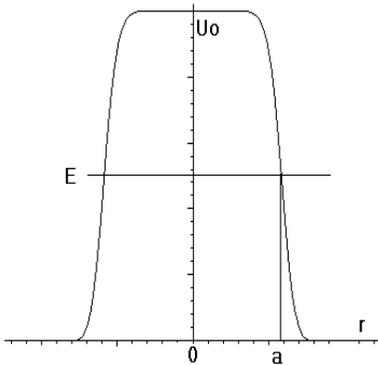
## 2.11 Scattering of Particles on Short-Range Potential (Potentials of Yukawa Type)

*“I recollect our discussions with Bohr. At the end of one of them I walked in the nearest park and asked myself once and again the same question: whether it is possible the nature being so absurd as we fancy in our atomic experiments”.*

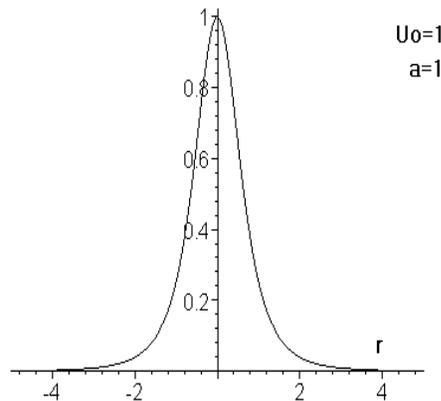
*Werner Heisenberg*

Usual scattering of particles on Coulomb potential at different angles corresponds to monotonic dependence on particle’s velocity. However, if potential is short-range (for example like Yukawa one), then the scattering

maximums appear in different angles and scattering now has resonance character. The first deeply studied phenomenon of such a type was Ramsauer-Townsend effect. At the first years of quantum theory development that phenomenon attracted everybody interest. There was experimentally detected the abnormal large-scale penetration of gas molecules or atoms for low-velocity electrons. In more general sense there was discovered non-monotonic dependence of effective cross-section of low-velocity electrons scattering on their velocity. Such dependence was at deep contradiction with classical idea, because according to it scattering monotonically decreased with electron velocity growth. But it were appeared for Ar, Kr, Xe that with the energy growth the effective cross-section of scattering run up to its maximum near 12 eV and then smoothly decreased. Deep minimum of full efficient section was in the area of energies of range 0.7 eV. Later the same effect had been proved within researches of electron mobility in gas. The length of electron free path was calculated on the basis of measuring. This length with velocity growth from  $4 \cdot 10^7$  up to sm./sec decreased abruptly.



**Fig. 2.11.1** Short-range potential.



**Fig. 2.11.2** Polynomial potential.

Let have a look at that effect from the viewpoint of standard quantum theory. Assume that scattering takes place on arbitrary steeply dropping potential with

strong repulsive core. In Fig. 2.11.1 values  $U_0$  and  $a$  are barrier height and width correspondingly, and the energy of the approaching particle  $E \ll U_0$ . At potential boundary we have  $\Psi(a) = 0$ . In the spherically symmetric case at  $r > a$  the wave function  $\Psi$  of the particle will be written as sum of incident and divergent spherical wave:

$$\Psi = \exp(ikz) + f(\theta) \frac{\exp(ikr)}{r}$$

where  $f(\theta)$  is the scattering amplitude. Usual flat wave contains spherically symmetric part that results from its expansion in polynomials of Legendre:

$$\exp(ikz) = \exp(ikr \cos(\theta)) = \sum_{l=0}^{\infty} f_l P_l(\cos(\theta)).$$

Then

$$f_0 = \frac{1}{2} \int_0^\pi \exp(ikr \cos(\theta)) \sin(\theta) d\theta = \frac{\sin(kr)}{kr}$$

and

$$\int \Psi \frac{d\Omega}{4\pi} \Big|_{r>a} = \frac{\sin(kr)}{kr} + \frac{f}{r} \exp(ikr)$$

By using boundary condition  $\Psi(a) = 0$ , we obtain:

$$\frac{\sin(ka)}{k} + f \exp(ika) = 0$$

Now we can write  $f$  in the form of

$$f = -\frac{\sin(ka)}{k} \exp(-ika).$$

At energies of the particle to be equal  $E = \frac{\hbar^2 \pi^2}{2a^2} n$ , and if  $n$  is an integer, then the resonance effect is evident. In these cases the cross-section of scattering equals zero. But in the case of repulsive potential the effect relaxes by participation of waves with  $l \neq 0$  in the processes of scattering at  $ka > 1$  and so full cross-section differs from zero. The biggest cross-section is obtained provided  $ka = (2n+1)\frac{\pi}{2}$ ,  $\sigma = 4\pi\hbar^2$  and at small values of  $k$  ( $ka \ll 1$ ), the cross-section is equal to  $\sigma = 4\pi a^2$ . The similar resonance phenomena are well known in optics – enlightenment of lenses for optical devices. For that the surface of the lens is covered with the special film of such a thickness and the index of refraction to obtain such a difference in phases of waves reflected from film and glass that the waves are able to suppress each other. In that case the reflected wave does not exist at all. That effect is similar to full transparency of one-dimensional barrier at definite energies considered in sect. 2.6 above.

In general, the mathematical modeling of considered processes with the help of the equations with oscillated charge has confirmed the existence of the phenomena said above. It remains valid for the equations with oscillating charge too. Since the use of potentials containing exponents (for example, of the Yukawa or Gauss potentials) have led to computational difficulties because of their rapid grow and possible overflowing with further stopping of calculations, we have used the following spherically symmetric polynomial potential (see Fig 2.11.2):

$$U(r) = \frac{U_0}{\left(1 + \left(\frac{r}{a}\right)^2\right)^2},$$

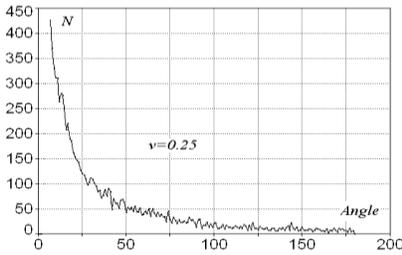
Assume that the immovable source of scattering potential  $U(r)$  is

time-constant and is placed at the origin of fixed coordinate system OXY. Then the non-autonomous system of equations with oscillating charge describing the particles' motion on coordinate plane OXY has the form [195]:

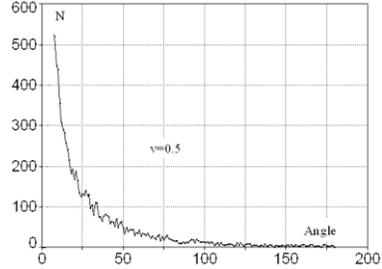
$$m \frac{d^2x}{dt^2} = \frac{8QU_0a^4x}{(a^2 + x^2 + y^2)^3} \cos^2 \left( \frac{mt}{2\hbar} \left( \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \right) - \frac{m}{\hbar} \left( x \frac{dx}{dt} + y \frac{dy}{dt} \right) + \varphi_0 \right),$$

$$m \frac{d^2y}{dt^2} = \frac{8QU_0a^4y}{(a^2 + x^2 + y^2)^3} \cos^2 \left( \frac{mt}{2\hbar} \left( \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \right) - \frac{m}{\hbar} \left( x \frac{dx}{dt} + y \frac{dy}{dt} \right) + \varphi_0 \right),$$

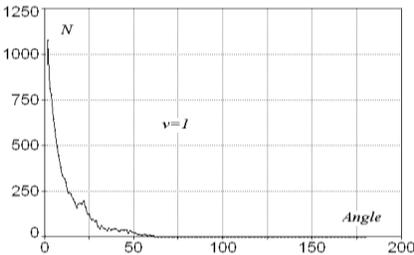
where  $m$ ,  $Q$  are mass and charge's constant part of particle. In non-autonomous case the first term of cosine argument is absent. We have put, for the sake of simplicity,  $Q = m = a = \hbar = 1$ ,  $U_0 = 5$ , and these equations were solved numerically by Runge-Kutta-Fehlberg method under initial values  $x(0)=100$ ,  $\dot{x}(0)=0$ , different initial  $y(0) \in (0,6)$  (sighting distances), and different initial velocities  $v_0 = \dot{y}(0) = 0.25, 0.5, 1, 2, 4$ . Calculations were stopped when the values of  $|x|$  or  $y$  for outgoing particle reached 100 and the scattering angle was computed by formula  $\theta = \arctg(\frac{\dot{y}}{\dot{x}})$  if  $\dot{x} > 0$  or  $\theta = \pi - \arctg(\frac{\dot{y}}{\dot{x}})$  if  $\dot{x} < 0$ . We have derived five curves (Fig. 2.11.3 – 2.11.7) expressing the relations between the scattering angle  $\theta$  and the quantity  $N$  of particles having given  $\theta$ . Each of these curves is based on 10000 trajectories corresponding to random, uniformly distributed initial phases with range  $0 - \pi$ , to sighting distances in interval  $(0, 6)$ , and to different above mentioned initial velocities. At some of them we can clearly see the influence of resonance effects.



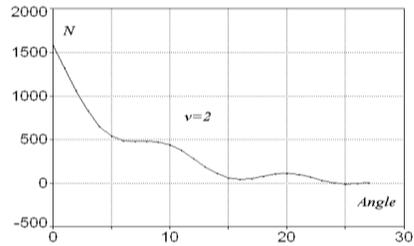
**Fig. 2.11.3** The scattering angle  $\theta$  and the quantity  $N$  of particles for velocity  $v=0.25$ .



**Fig. 2.11.4** The scattering angle  $\theta$  and the quantity  $N$  of particles for velocity  $v=0.5$ .



**Fig. 2.11.5** The scattering angle  $\theta$  and the quantity  $N$  of particles for velocity  $v=1$ .



**Fig. 2.11.6** The scattering angle  $\theta$  and the quantity  $N$  of particles for velocity  $v=2$ .

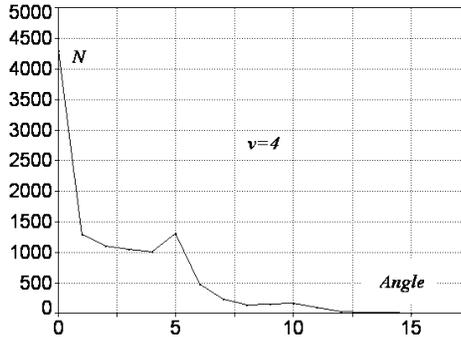
The obtained curves allow to affirm that at small energies the character of scattering starting from angles about 50 degrees does not depends almost on angle (almost isotropic scattering), but in the area of small angles differential cross-section is distinctly anisotropic. One can see that with the increase of energy the scattering is manifested in the areas of more and more reducing angles. Rather crude estimation of these angles can be obtained from the following considerations: in the case of interaction of the fast particle, for example, of nucleon having kinetic energy  $T \gg U$ , with other nucleon its impulse is able to change at value not more then  $\Delta P$ , related with well depth  $U$  as follows:

$$\frac{\Delta P^2}{2m} = U$$

Now we can obtain the scattering angle  $\theta$  :

$$\theta \approx \frac{\Delta P}{P} = \sqrt{\frac{2mU}{2mT}} = \sqrt{\frac{U}{T}} = \sqrt{\frac{25}{400}} = 0.25 \approx 15^\circ \quad (2.11.1)$$

That consideration should not be taken as inconsistency because all violations of conservation law should take place at very small energies.



**Fig. 2.11.7** The scattering angle  $\theta$  and the quantity  $N$  of particles for velocity  $v=4$ .

It can be easily seen from the relation (2.11.1) that the scattering angle is inversely proportional to velocity. We can easily get the same result at the obtained numeric data.

The processes with looking like diffraction scattering begin with the growth of energy. In general theory of diffraction scattering it is studied the scattering on opaque or gray disk or sphere. According to Babine principle, diffraction pattern caused by disk (potential barrier) coincides with diffraction pattern caused by screen with whole (potential well) to be equal in dimensions to the disk. It is astonishing but potential sign inversion (change of barrier for the potential well) does not practically influence curves of differential cross-section, although it cannot be considered as strict mathematical fact because equations are nonlinear.

In Fraunhofer approximation the amplitude of scattering and curve of

differential effective cross-section (diffraction was studied on black disk with  $R_0$  radius) are described by Bessel functions:

$$f(\theta) = ikR_0^2 \frac{J_1(kR_0\theta)}{kR_0\theta}$$

$$\frac{d\sigma}{d\Omega} = (kR_0^2)^2 \left( \frac{J_1(z)}{z} \right)^2, \quad (2.11.2)$$

where  $z = kR_0\theta$ . The first null of the function  $J_1(z)$  arise at value  $z=3.84$ . Then the first minimum of diffraction pattern appears at

$$\theta_{\min} = \frac{3.84}{kR_0} = 0.61 \frac{\lambda}{R_0} = 0.61 \frac{h}{mvR_0}$$

The location of the first minimum at scattering curve is inversely proportional to velocity  $v$ .

Classical approach has one more specific feature. Viz., as far

$$\lim_{z \rightarrow 0} [J(z)] = \frac{z}{2},$$

differential effective cross-section of elastic scattering forward will be following

$$\frac{d\sigma(\theta^0)}{d\Omega} = \frac{1}{4} k^2 R_0^4 \quad (2.11.3)$$

It may be seen that with tending of falling particle energy to infinity the differential cross section of scattering in direction of the angle  $\theta^0$  increases as  $k^2$ .

But till now it is the viewpoint of classical waves physics only. In physics of

elementary particles the same conclusions will be written in more usual form. In system with origin in mass center of bumping particles the transmitted momentum  $q$  at small angles of elastic scattering is written in the form:

$$|\mathbf{q}| = 2\hbar k \sin\left(\frac{\theta}{2}\right) \approx \hbar k \theta$$

In standard symbols

$$-t \equiv q^2$$

Then

$$-dt = 2\hbar^2 k^2 \theta d\theta$$

Equation (2.11.2) can be rewritten

$$\frac{d\sigma}{dt} = -\frac{\pi}{\hbar^2 k^2} \frac{d\sigma}{d\Omega} = \frac{\pi R_0^4}{\hbar^2} \frac{J_1^2\left(\sqrt{\frac{tR_0^2}{\hbar^2}}\right)}{\frac{tR_0^2}{\hbar^2}}$$

And then equation (2.11.3) can be rewritten in the form:

$$\frac{d\sigma}{dt}(t=0) = -\frac{\pi}{4\hbar^2} R_0^4$$

So,  $\frac{d\sigma}{dt}$  depends on  $t$ , i.e. on square of transmitted momentum only, but not on the energy of incident particle, and at  $t=0$  it does not depend on momentum at all.

Astonishing is the fact of approximate coincidence of the first minimum of the scattering curve offsets depending on velocity. Thus at curve in Fig. 2.11.5 the first minimum arises at 16 degree. At Fig. 2.11.6 first minimum is near 8 degree

that answers two times changing of the velocity. The more impressive comparison gives Fig. 2.11.6 and Fig. 2.11.7, where minimums' and maximums' offsets submit to equation (2.12.2) and detect regularity that scattering angle is inversely proportional to velocity. We should admit that we have not expected such results at all because of the current opinion that diffraction scattering can be seen only in the cases when strong inelastic interaction presents and scattering particles wave length is small in comparison with radius of interaction (neutrons scattering on nuclei and pions scattering on nucleons). We should note that in accordance with the strict Unitary Quantum Theory the division of scattering processes into elastic and inelastic ones is a kind of idealization. That conclusion can be also extended at equations with oscillating charge. More over the most amazing is the fact of discovery of wave character of the mention process described by non-linear equations.

It was carried out also the modeling of particles' scattering on some others potentials of Yukawa or Gauss types and the general pattern of the processes were the same. The only difference was the more sharp appearance of resonance peaks at higher energies.

We have carried out the same calculations in the case of the autonomous equations. The results are practically congruent with the above-mentioned ones. We are not going to analyze large quantity of experimental data dealing with differential cross-sections of various scattering processes. It is a problem for future. That is why we even have not integrated the differential cross-sections to get the full picture.

The approach examined can be easily extended to the scattering caused by object consisting of few connected particles (Glauber approximation).

In general there is nothing strange or new in such pictures of scattering.

In unitary quantum theory mass spectrum of numerous elementary particles were obtained [162, 181]. It appeared that in terms of mass density any particle can be presented as a bubble parted by spherical harmonic. For simplicity the heaviest Dzhan particle can be presented as a potential Fig. 2.11.8 [195]:

$$U(r) = r^2 e^{-r^2}$$

Our system of equations would look like:

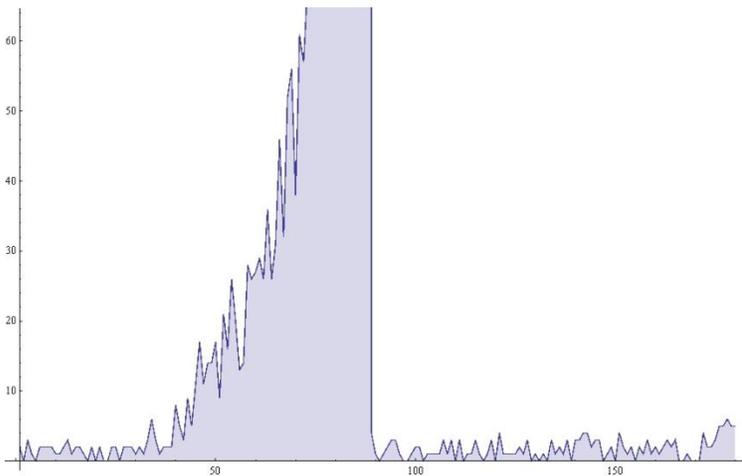
$$\frac{d^2x}{dt^2} = 2Q \frac{(1-x^2-y^2)x}{e^{x^2+y^2}} \cos^2\left(-x \frac{dx}{dt} - y \frac{dy}{dt} + \varphi\right) \quad (2.11.4)$$

$$\frac{d^2y}{dt^2} = 2Q \frac{(1-x^2-y^2)y}{e^{x^2+y^2}} \cos^2\left(-x \frac{dx}{dt} - y \frac{dy}{dt} + \varphi\right) \quad (2.11.5)$$

For initial values

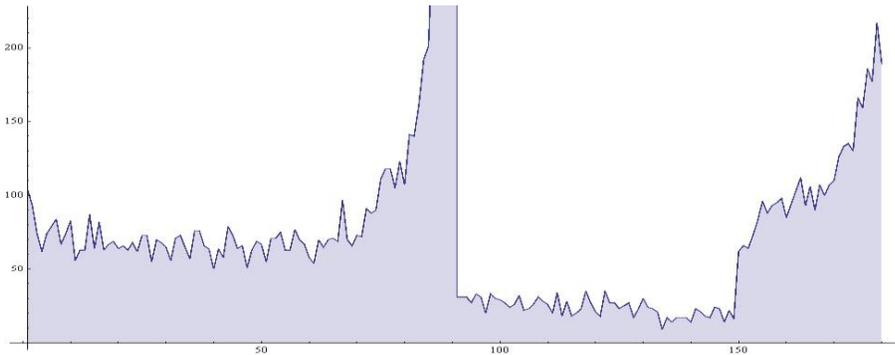
$$Vx_0 = \frac{1}{3}, x_0 = -1000, b = 0 \div 100, \\ \varphi = 0 \div \pi, Vy_0 = 0, Q = 5, \Delta T = 3000, N = 10000$$

can see Fig. 2.11.8



**Fig. 2.11.8** Dependence of the Scattered Particles Number from the Angle for Eq.2.11.4, 2.11.5.

If we change slightly the velocity and take  $Vx_0=0.3344$  then scattering change sharply (Fig. 2.11.9). Obviously striking resonant phenomena appears inside the bubbles. It looks like hadronic streams, but



**Fig. 2.11.9** *Dependence of the Scattered Particles Number from the Angle for the Eq.2.11.4, 2.11.5.*

this diagram does not concern that phenomenon anyhow. Probably it can be observed at high energies but it's a question of far future. The method discussed here can be helpful in future for the construction of scattering potentials corresponding to different scattering pictures.

We should admit that we have not expected such results at all because of the current opinion that diffraction scattering can be seen only in the cases when strong inelastic interaction presents and scattering particles wave length is small in comparison with radius of interaction (neutrons scattering on nuclei and pions scattering on nucleons). We should note that in accordance with the strict Unitary Quantum Theory the division of scattering processes into elastic and inelastic ones is a kind of idealization [195]. That conclusion can be also extended at equations with oscillating charge. More over the most astonishing is the fact of discovery of wave character of the mention process described by non-linear equations. In general, the results of mathematical modeling are coinciding with

intuitively expected ones on the base of qualitative analysis. For example, Ramzauer-Townsend effect is totally understandable. Really, if the length of de Broglie wave is much bigger than atom dimensions, and if the incident electron has the phase corresponding to very small charge, then appears the effect of irregular atom transparency. These electrons pass through atom. Similar situation exists in the s-state of the atom [172, 183, 200, 201].

As de Broglie wave length is very slowly decreases with the energy, such effects of high transparency should take place for any particles at head-on collision. It is evident from the analysis of experimental data that in modern physics the said effect is detected for any particles of counter-current bunches at head-on collision.

It was carried out also the modeling of particles' scattering on some others potentials of Yukawa or Gauss types and the general pattern of the processes were the same. The only difference was the more sharp appearance of resonance peaks at higher energies.

We have carried out the same calculations in the case of the autonomous equations. The results are practically congruent with the above-mentioned ones. We are not going to analyze large quantity of experimental data dealing with differential cross-sections of various scattering processes. It is a problem of future. That is why we even have not integrated the differential cross-sections to get the full picture.

The approach examined can be easily extended to the scattering caused by object consisting of few connected particles (Glauber approximation).

## 2.12 Particle with Oscillating Charge and Periodic Chain of Barriers

*Well, he has breached the blank wall he was up against.*

*And now what will he do in that next-door ward?*

*“Shaggy thoughts” Stanislav Ejy Letz*

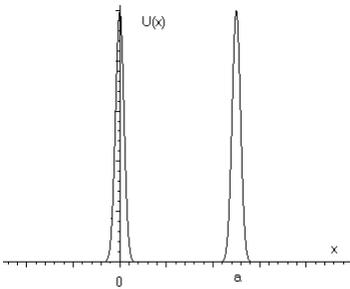
Interesting phenomena may be observed in the case of potential barriers series. From the pure qualitative UQT positions it is evident that if there are two high but quite narrow potential barriers situated at some distance one from another, then the first barrier will be penetrated by those particles only which phase is so that at the moment of the first barrier reaching the particle charge is very small. In that case the particle will pass the first barrier. The second barrier will be also is passed by the particles having in front of the barrier again the phase corresponding to the very small charge. Such a system of two or more periodic barriers results in the fact that a monochromatic correlated in phases flow will be cut out of the particles flow with various energies and phases. In cross- section of that flow there will be particles in one phase only. All this will look like the military favorable training: soldiers are marching, keeping the step and its dimension for all soldiers is strictly equal.

The same considerations relative to barrier's chain exists within standard quantum mechanics but from the viewpoint of that theory one can say nothing about the wave's phase and about the physical sense of observed phenomena. Let us consider in details that quite interesting situation [172, 183, 200, 201]. First clear up how it happens in accordance with standard quantum mechanics.

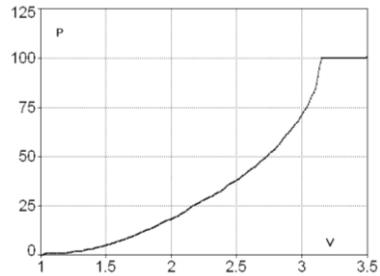
### 2.12.1 Two Barriers

Consider the problem of particle's passing through the system of two potential barriers described by Dirac's unit-impulse functions and situated at some distance  $a$  one from the other. The potential of such system is following (Fig. 2.12.1):

$$U(x) = a[\delta(x) + \delta(x - a)]$$



**Fig. 2.12.1** Potential of two-barriers system.



**Fig. 2.12.2** The numbers of particles (percentage  $p$ ) passing two barriers in respect to particles velocity (autonomous equation).

Assume the particle's flow moving from left to the right. Let's determine the particle's energy  $E$  required for passing both barriers. The Schroedinger equation for the wave function is following:

$$-\frac{\hbar^2}{2m} \Psi'' + a[\delta(x) + \delta(x - a)]\Psi = E\Psi \tag{2.12.1}$$

At once we can write its solution for the area 1 ( $x < 0$ ) before the barrier, where according to common approach the incident wave exists only. The solution for the area 2 ( $0 < x < a$ ) between the barriers contains both right and reversed waves. The solution for the area 3 ( $x > a$ ) behind the second barrier contains the passed wave only. Therefore, we have the following solutions:

$$\Psi_1(x) = \exp(ikx), \quad x < 0, \quad k = \sqrt{\frac{2mE}{\hbar^2}} > 0,$$

$$\Psi_2(x) = A \sin(kx) + B \cos(kx) \quad 0 < x < a,$$

$$\Psi_3(x) = C \exp[ik(x-a)] \quad x > a,$$

The continuity of the wave function and discontinuous character of the derivative in points  $x=0, x=a$  leads to the equalities:

$$\Psi'(+0) - \Psi'(-0) = \frac{2ma}{\hbar^2} \Psi(0)$$

$$\Psi(+0) = \Psi(-0)$$

Joining in a standard way the wave functions and their derivatives in the points  $x=0$ , and  $x=a$  and taking into account the above equalities, we get the system of four algebraic equations:

$$B = I$$

$$kA - ik = \frac{2ma}{\hbar^2}$$

$$A \sin(ka) + B \cos(ka) = C$$

$$ikC - kA \cos(ka) + kB \sin(ka) = \frac{2maC}{\hbar^2} \quad (*)$$

The given system is predefined and has solution only under following condition:

$$\operatorname{tg}(ka) + \frac{\hbar^2 k}{ma} = 0.$$

---

\* We have obtained (see sect.2.6) the full and mathematically rigorous solution of discussed problem. The results are, generally speaking, another but they lead to the analogues conclusions.

If  $k_1, k_2, \dots$  are the roots of this equation, then using the expression for  $k$  (written at the beginning of this sect.), we are able to determine the energy values at which a particles penetrate (we say, tunnels) two-barrier's system:

$$E_s = \frac{\hbar^2 k^2}{2m}, s = 1, 2, \dots,$$

It is evident from the solution of transcendental equation that periodic dependence in energy while tunneling two barriers appears because of tangent curve that have be periodically crossed by straight line emergent on some angle from the origin of coordinate system. It is evident that barriers will be passed by the particles with de Broglie wavelength being multiple to  $a$ . That phenomenon bears a strong resemblance to processes appearing in the cases of antireflecting optic lenses.

We should note an interesting circumstance. If the same problem were solved in other order, i.e. to determine first the portion of the particles flux penetrated (tunneled) the barrier and to consider the passed portion as incident flux in respect to the second barrier, the result would be absolutely different. The multiplication of two exponents to be given by each barrier just suppresses everything. It is very difficult to understand such double game directive for an unprejudiced physician with mentality non-perverted by “quantum” logic.

There is one more amazing consideration. Assume the particle does not penetrate the barrier but just going to tunnel it or to be reflected, but it “decision” depends on the distance to the second barrier. But how could it know what will be happened and what is the distance to the second barrier. Does the second barrier exist at all? Here we can recollect the perfect words of R. Feynman. May be the particle “sniffs out” the second barrier? And again violence over logic and mind.

Similar phenomena but in more tangible and totally understandable form takes place if we analyze the solutions of the equation with oscillating charge.

To our regret numerical modeling is embarrassing in the case of barriers system (2.12.1) described by delta functions, and that is why we have replaced (2.12.1) by the sum of two Gauss “bells”:

$$U(x) = U_0 \left[ \exp\left(-\frac{x^2}{\sigma^2}\right) + \exp\left(-\frac{(x-a)^2}{\sigma^2}\right) \right]$$

The one-dimensional equation with oscillating charge describing the particle’s motion and corresponding to last potential were solved numerically in autonomous and non-autonomous cases. As far as the results obtained are slightly different we show them separately with further comparison.

### 2.12.2 Autonomous Model

The one-dimension equation of motion has the following form:

$$m \frac{d^2 x}{dt^2} - 4U_0 Q \left[ x \exp\left(-\frac{x^2}{\sigma^2}\right) + x \exp\left(-\frac{(x-a)^2}{\sigma^2}\right) - a \exp\left(-\frac{(x-a)^2}{\sigma^2}\right) \right] \frac{\cos^2(\varphi)}{\sigma^2} = 0, \quad (2.12.2)$$

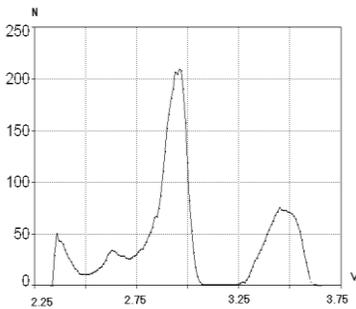
where

$$\phi = -\frac{m}{\hbar} \frac{dx}{dt} x + \phi_0, \quad (2.12.3)$$

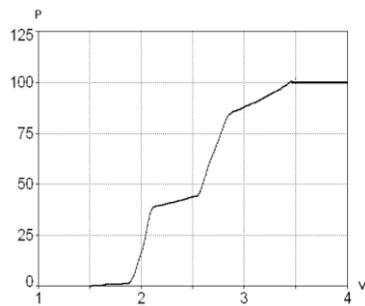
$x = x(t)$ ,  $\varphi_0$  -initial phase, a-distance between the barriers,  $\sigma, U_0$  are the width and height of barrier respectively, Q, m are the constant part of particle’s charge and mass respectively. The equation was solved numerically by Runge-Kutta-Merson method. The numbers of particles tunneled with respect to initial velocity and to initial phases uniformly distributed in interval from 0 to  $\pi$

were calculated. The following starting data were used:  $Q=1$ ,  $m=1$ ,  $\hbar=1$ ,  $U_0=5$ ,  $a=4$ ,  $\sigma=1/8$ ,  $V_0=1-3.5$ . For each initial velocity value we computed variants for 101 values of initial phase (the case of  $\phi_0 = \frac{\pi}{2}$  was excluded from calculations). The total number of the particles equals 20502. The results of calculations are shown in the Fig. 2.12.2, Fig. 2.12.3. The relation between numbers (percentage) of particle and the initial velocity (Fig. 2.12.2) can be well approximated by simple exponent. The distribution of particle's number in respect to the velocity after passing two barriers (Fig. 2.12.3) does not show a resonance effect.

From the other side, particles' grouping in respect to velocity exists as it was described in the beginning of that section and expected from the most general considerations. It can be seen in Fig. 2.12.3, where x-axis indicates particles' velocities and y-axis indicates the number of particles. It is curious that within the velocities interval (1, 2.27) and (3, 3.25) there is no particles at all (forbidden zones). We will discuss it in details later.



**Fig. 2.12.3** Distribution of particles in respect to velocity after passing two barriers (autonomous equation).



**Fig. 2.12.4** Probability of passing two barriers in respect to velocity of particles (non-autonomous equation).

### 2.12.3 Non-autonomous Model

Non-autonomous equation has the same form (2.12.2), but with the other expression of  $\varphi$  :

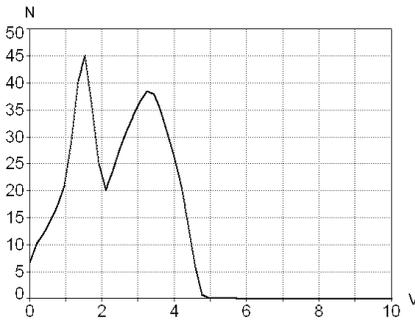
$$\phi = \frac{mt}{2\hbar} \left( \frac{dx}{dt} \right)^2 - \frac{m}{\hbar} \frac{dx}{dt} x + \phi_0 \quad (2.12.4)$$

The numerical integration was made by Runge-Kutta-Merson method using following data:  $Q=1$ ,  $m=1$ ,  $\hbar = 1$ ,  $U_0 = 5$ ,  $a=4$ ,  $\sigma = 1/8$ ,  $V_0 = 1.5-4$ . The full number of particles  $N=20502$ . Now, as it can be seen in Fig. 2.12.4, the expected resonant dependence of barrier's tunneling on the initial velocity exists. But that resonance effect is slightly suppressed because tunneling probability is increasing with the velocity thus compensating the drop in probability at moving away from resonance point (horizontal steps at curve). We have plotted particles' exit velocity distribution after tunneling (Fig. 2.12.5). It is evident from the plot that the forbidden zones within the area of velocity equal to 2 are outlined and also may be seen the particles' grouping in respect to velocities.

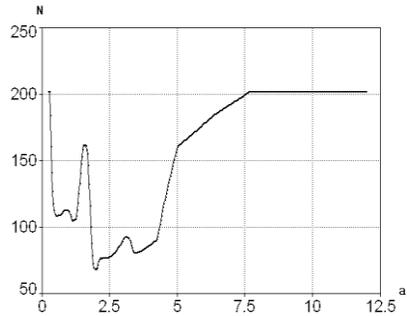
We calculated also with the help of numerical integration of the same equation the probability of barrier tunneling with respect to distance between the barriers  $a$ , to fixed initial velocity of particles, and to the initial phases uniformly distributed from 0 to  $\pi$ . Starting data were the following:  $Q=1$ ,  $m=1$ ,  $\hbar = 1$ ,  $U_0 = 5$ ,  $a=0.25-12.5$ ,  $\sigma = 1/8$ ,  $V_0 = 2.5$ .

Total number of particles  $N=20502$ . The obtained dependence may be estimated as expected from quite general viewpoint. Really, the first barrier is to be passed by all particles which phase is so that the particle charge in front of the barrier is small. If further on particle's way the second barrier were appeared,

then it would be penetrated without any problem by particles with the phase corresponding to the small charge in front of the barrier again. If the distance between the barriers is variable, the tunneling effect will have periodical character. That is illustrated in Fig. 2.12.6.



**Fig. 2.12.5** Distribution of particles in respect to velocities after passing two barriers (non-autonomous equation).



**Fig. 2.12.6** Number of particles passing the barriers in respect to distance between barriers (non-autonomous equation).

In general the character of solutions for both types of equations is similar, there are appeared the forbidden zones, but the fact that one can see both increasing and decreasing velocity in comparison with initial velocity is more interesting. Thus, in Fig. 2.12.3 it can be seen that there are many particles with velocity a little bit more than initial maximum velocity equal to 3.5. We can see the same phenomenon in Fig. 2.12.5, where the number of particles with the velocity more than 4 – initial maximum velocity is enough. It is also evident that in both cases there are many particles with velocity less than the minimal initial one. In any case we do not think that some particles give its energy to other particles and reduce its velocity and visa versa what is a cause of Maxwell-Boltzmann statistical distribution. The modeling is made each time for one particle only (!) and equation does not mean any interaction with the other particles. We would like to think that additional energy of obtained ensemble owing to fast particles is just exactly equal to the energy lost by slow ones. Of course it is pure aesthetic consideration which etymology descends

from atavistic nostalgia in conservation law, but we have not checked this circumstance. Besides, such reasons are based on energy and momentum conservation laws that are not fulfilled for both equations.

### 2.12.4 Three Barriers

Let's clear up the particle's behavior in the case of a periodical potential according to standard quantum mechanics. We will use the general perturbation theory and examine the changes of free particle motion caused by perturbing periodic potential

$$V = V_0 [\exp(ikx) + \exp(-ikx)]$$

The wave function for free undisturbed motion equals  $\Psi_p = \exp(ikx)$ . Average value  $V$  over any undisturbed state equals zero; following matrix elements differ from zero only:

$$V_{p,p-k} = V_{p,p+k} = V_0$$

Let us consider the length of potential hole equal to 1. According to standard perturbation theory energetic level shift equals

$$E_p = \varepsilon_p + \frac{V_0^2}{\varepsilon_p - \varepsilon_{p-k}} + \frac{V_0^2}{\varepsilon_p - \varepsilon_{p+k}}, \text{ where } \varepsilon_p = \frac{p^2}{2m}.$$

Of course that expression will be true, provided

$$V_0 \ll |\varepsilon_p - \varepsilon_{p-k}|,$$

when  $p$  is far from  $\pm \frac{k}{2}$ . If  $p \rightarrow \frac{k}{2}$ , then states  $\Psi_p$  and  $\Psi_{p-k}$  possess close values of energy, and wave function is to be sought in the form

$$\Psi = C_1 \Psi_p + C_2 \Psi_{p-k}.$$

If  $p \rightarrow -\frac{k}{2}$ , then states  $\Psi_p$  и  $\Psi_{p+k}$  have near values of energy again and wave function is to be sought in the form

$$\Psi = C_1 \Psi_p + C_2 \Psi_{p+k}.$$

Basing on the perturbation theory [19], we get following proper values of energy:

$$E_p = \frac{\varepsilon_p + \varepsilon_{p-k}}{2} \pm \sqrt{\frac{(\varepsilon_p - \varepsilon_{p-k})^2}{4} + V_0^2}. \quad (2.12.5)$$

The sign in expression (2.12.5) is determined by the condition that  $E_p \rightarrow \varepsilon_p$  if

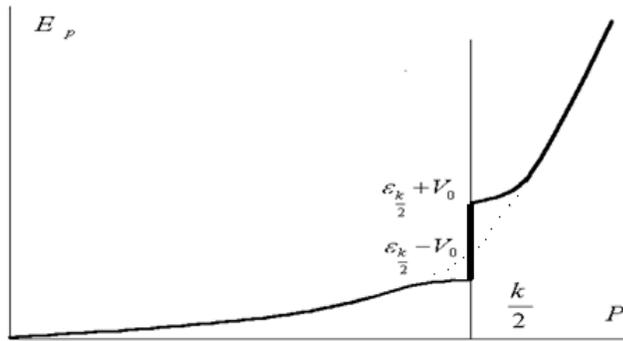
$|\varepsilon_p - \varepsilon_{p-k}| \gg V_0$ . Viz., if  $p < \frac{k}{2}$ , then

$$\sqrt{(\varepsilon_p - \varepsilon_{p-k})^2} = |\varepsilon_p - \varepsilon_{p-k}| = \varepsilon_{p-k} - \varepsilon_p,$$

and if  $p > \frac{k}{2}$ , then  $\sqrt{(\varepsilon_p - \varepsilon_{p-k})^2} = \varepsilon_p - \varepsilon_{p-k}$ . Hence, for  $p < \frac{k}{2}$  one sign is to be taken and for  $p > \frac{k}{2}$  the other. For  $E_p$  we obtain the well-known curve having the jump (Fig. 2.12.7). The value of the jump is equal

$$E_{\frac{k}{2}+0} - E_{\frac{k}{2}-0} = 2V_0.$$

Thus, in the case of periodical potential the spectrum of free particles has an energy forbidden zone equal to  $2V_0$ .



**Fig. 2.12.7** *Appearing of energetic jump Further, there was modeling the particles behavior while tunneling three Gauss “bells”. Plot of potential is shown in Fig. 2.12.8.*

The expression for the three - barriers potential function is following:

$$U(x) = U_0 \left[ \exp\left(-\frac{x^2}{\sigma^2}\right) + \exp\left(\frac{(x-a)^2}{\sigma^2}\right) + \exp\left(-\frac{(x-b)^2}{\sigma^2}\right) \right]$$

The equation of particles' motion has the form:

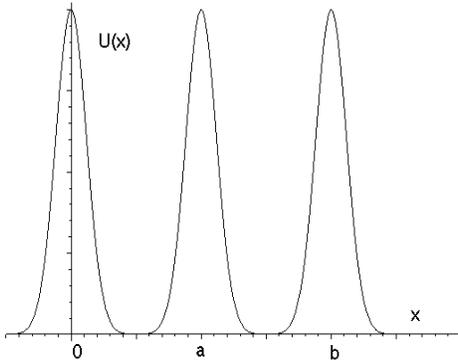
$$m \frac{d^2x}{dt^2} - \frac{4U_0Q}{\sigma^2} \left[ x \exp\left(-\frac{x^2}{\sigma^2}\right) + (x-a) \exp\left(-\frac{(x-a)^2}{\sigma^2}\right) + (x-b) \exp\left(-\frac{(x-b)^2}{\sigma^2}\right) \right] \cos^2(\phi) = 0,$$

where

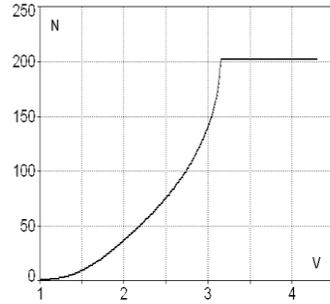
$$\phi = -\frac{mx}{\hbar} \frac{dx}{dt} + \phi_0 \text{ in the autonomous case}$$

and

$$\phi = \frac{m}{2\hbar} \left(\frac{dx}{dt}\right)^2 - \frac{mx}{\hbar} \frac{dx}{dt} + \phi_0 \text{ in the non-autonomous case.}$$



**Fig. 2.12.8** Potential of three barriers.



**Fig. 2.12.9** Number of passed particles in respect to initial velocity.

Particles are flying from left towards the barriers from distance more than  $4a$  along  $x$ - axis. The equation was integrated numerically in autonomous and non-autonomous cases by using Runge-Kutta-Merson method. If there had been oscillation appearing between the barriers (velocity sign changing) then such particles were eliminated. The initial data were following:

$$U_0 = 5, \sigma = 1/8, \hbar = 1, Q = 1, m = 1.$$

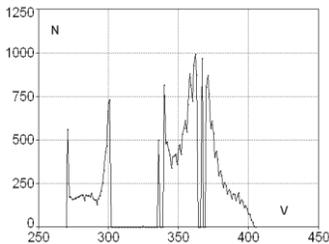
There were chosen 201 different values of initial phase for each value of velocity, and intervals of velocities (from 1 to 4.3 in autonomous case and from 1.6 to 4.5 in non-autonomous case) were divided by 400. Thus, the motion of more than 80000 particles was analyzed for each case.

### 2.12.5 Autonomous Case

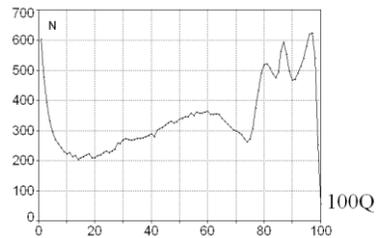
There is shown in Fig. 2.12.9 the number of particles passed all the barriers in respect to velocity in autonomous case, but it is not detected the expected periodic dependence of tunneling coefficient on velocity. The character of the curve corresponds to the dependence in the case of two barriers. The curve itself

is very well approximated by exponent. There are represented in Fig. 2.12.10 and Fig. 2.12.11 distributions of particles' number in respect to velocities and charges after passing three barriers.

The distribution in respect to velocities shows the presence of 6 evident forbidden zones. It can be also seen that width and the number of forbidden zones are increasing with the growth of the barriers' number. However, width of the zones in the cases of 2 and 3 barriers problems is not equal to  $2U_0$ . Apparently, it is because of small number of barriers.



**Fig. 2.12.10** Distribution of particles' number in respect to velocity after passing 3 barriers.

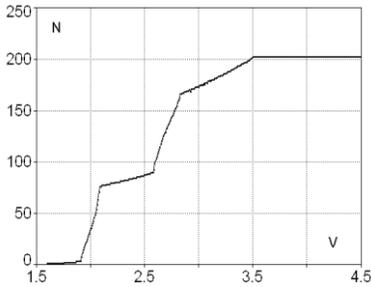


**Fig. 2.12.11** Distribution of particles' number in respect to charge value after passing 3 barriers.

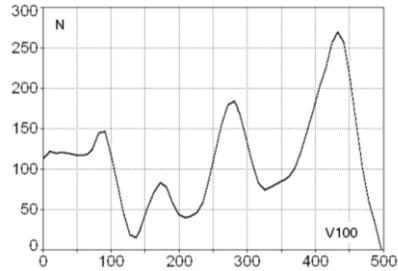
It may be said, that our expectations based on simple physical reasons are confirmed. The particles' grouping in respect to velocities and charges are intensifying in the case of three barriers in comparison with two barrier' case. However non-autonomous equation shows the other and possible more interesting results

## 2.12.6 Non-autonomous Equation

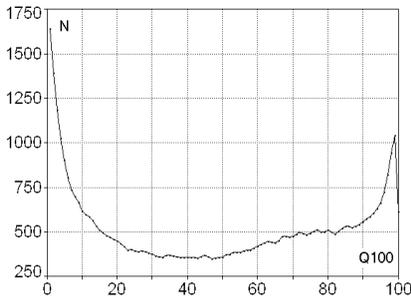
The plot for the number of passed particles in respect to the initial velocity is presented in Fig. 2.12.12.



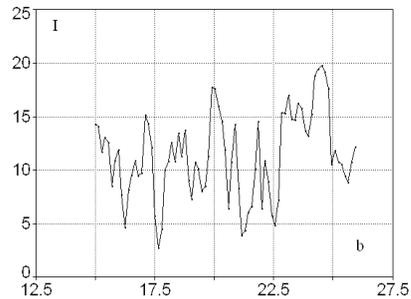
**Fig. 2.12.12** Number of passed particles in respect to velocities (non-autonomous case).



**Fig. 2.12.13** Distribution of particles' number in respect to velocities after passing of 3 barriers (non-autonomous case).



**Fig. 2.12.14** Distribution of particles number in respect to charges after passing 3 Barriers.



**Fig. 2.12.15** Electric current  $I$  as the function of coordinate  $b$  of third bar (non-autonomous case).

The plots in Fig. 2.12.4 and Fig. 2.12.13 are very similar. We can see the resonance dependencies at some values of velocity. At the other side, it can be seen in Fig. 2.12.14 outlining forbidden zones (curves minima), but they are not so strongly pronounced as the same in the case of autonomous case; besides, it is clear enough the depth and number of minima being increase in comparison with Fig. 2.12.5 (two-barriers case).

As the tunneling probability according to Unitary Quantum Theory depends on initial phase, the following nontrivial effect should take place: after two barriers tunneling by incident particles with different velocities and initial phases, it

appears the flow correlated in respect to velocities and charges. If now the third potential barrier were placed on the way of such correlated flow at such a distance that all approaching particles have the phase corresponding to small charge, then each particle would penetrate. But if we were moving the barrier nearer or farther, then the phase of each particle would be another and all particles should be reflected. In other words it may be outlined a fundamentally new possibility of extremely effective control of so correlated particles' flow. Such conclusions do not follow from standard quantum mechanics because according to it the initial phase is, so to say, unessential parameter. Note, the considered particles correlation in respect to velocities and charges looks in respect of the particles passed several barriers like output of photon flow from the laser. Additionally it offers, in principle, a series of perspective practical applications.

Further modeling of processes in the case of the three barriers was done using the algebraic potential as Gaussian potential restricts the effectiveness of numerical computation for sufficiently big values of variables. There was used the following potential corresponding to system of three barriers:

$$U(x) = U_0 \left[ \frac{1}{(1+x^2)^2} + \frac{1}{(1+(x-a)^2)^2} + \frac{1}{(1+(x-b)^2)^2} \right]$$

That equation of motion is following:

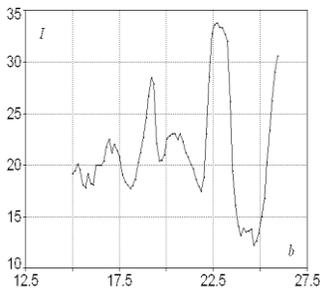
$$m \frac{d^2x}{dt^2} - 8QU_0 \left[ \frac{x}{(1+x^2)^3} + \frac{x-a}{(1+(x-a)^2)^3} + \frac{x-b}{(1+(x-b)^2)^3} \right] \cos^2(\varphi) = 0,$$

where  $\varphi$  is expressed by (2.12.3) or (2.12.4) for autonomous and non-autonomous cases respectively.

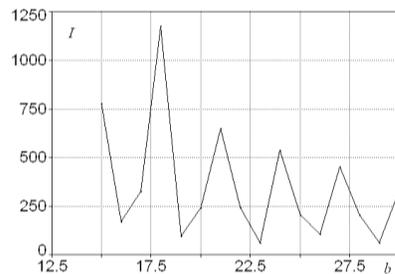
The results of numerical integration of last equation in autonomous and in

non-autonomous cases are presented in Fig. 2.12.15 and Fig. 2.12.16. The electric current in any fixed point is calculated as the product of particles' number, of velocity and charge.

It is important to notice that the value of electric current depends strongly on the distance to the third barrier. Fig. 2.12.15 shows the points, where the current is nearly vanishing. Such theoretical curves hardly may be obtained within standard quantum theory because particles charges, according to it, are strictly constant and the corresponding curves will be quite different.



**Fig. 2.12.16** Electric current  $I$  as function of coordinate  $b$  of third barrier (autonomous equation).



**Fig. 2.12.17** Electric current  $I$  as function of coordinate  $b$  of third barrier (nonautonomous equation).

But there is another effect appearing in the three-barrier problem that does not exist at all according to standard quantum mechanics. The wave function for a particle passed through three barriers has the form  $\Psi = \exp(ikx)$  and no current spatial oscillations arise after three barriers as  $\Psi \cdot \Psi^* = \text{Const}$ . But we can see in Fig. 2.12.17 the plot of estimated current value as function of distance to the third barrier (i.e. distance from barrier to particle passed already this barrier) and it may be easily seen the cyclic changes of electric current and some exponential fall of it along  $x$ -axis caused, apparently, by long polynomial tail of the third barrier

The same phenomenon may be observed in long experimental lines (so called,

Lechers wires). It is so popular that is shown in any respectable universities at lectures. During the experiment one can easily see that in so long lone there are points where the electric current is absent at all, and the other where tension equals to zero. During accurate demonstrations the line is cut in that points and insert thin isolating laying or is even abridged and nevertheless the lamp at the end of the line continues to shine. That phenomenon is often used for construction of modern ultrahigh frequency devices.

But it remains still incomprehensible how electrons penetrate through thick isolating laying, if the electric current is caused by motion of electrons? Conventional explanation is that wires of long line acts like a guide (like rails) in electromagnetic field and it is the field transmits the energy, not the wires. But then irremovable paradox appears: at extreme frequency reduction we will have to do with constant electric current and the energy of such current is transmitted by wires together with electrons flow.

Unitary Quantum Theory explains that paradox perfectly. Note that according to ordinary electrical and radio engineering the electron-drift velocity has the order of mm/sec. Then de Broglie electron's wavelength becomes a macroscopic value, in any case, many times more than the thickness of isolating laying used by experiments. We have to deal, in essence, with usual tunneling effect, but in macroscopic scale only. Nobody drew attention to such possible approach. The using of our results relative to barriers passage problem in the case of quantum wire accompanied by modern experimental data may help to make a choice in favor of Unitary Quantum Theory. More detailed analysis of that problem will be done in the section 3.6. dealing with quantum wires.

## 2.13 Uncertainty Relation and Principle of Complementarity Within UQT and Mechanics with Oscillating Charge

*I discard the main idea of modern statistic quantum theory... I do not believe such principal conception being an appropriate base for physics in general...I am quite sure that existing essentially statistic character of modern quantum theory should be ascribed to the fact that theory operates with incomplete description of physical systems.*

*A. Einstein*

As far as many nonsense have been announced concerning the uncertainty relation we would like to give more detailed of their obtaining first by W. Heisenberg then by N. Bohr and of not quite adequate their interpretation. So, Heisenberg derived the uncertainty relation on well-known way, now called the method of Heisenberg's microscope and based on the analysis of conditions when microparticle's position and motion can be experimentally detected. In principle, the particle's position can be determined by observations of light rays reflected, diffused or emitted by the particle. The particle is considered as a source of light and the results of its observation will be always the diffraction circle with radius equal to the wave length  $\lambda$  of this light rays. So the particle position can be determined with precision of order  $\lambda$ .

The most primitive idea to improve the accuracy of measurements is to use light rays with  $\lambda$  being as small as possible. We can use, for example, gamma sources, technical implementation of that idea for the time being is not so important. But at

the same time we faces A. Compton effect; in the process of measuring the gamma quantum is scattered by the particle and the impulse of the particle is changed for the value equal  $\frac{\hbar}{\lambda}$ . It is paradoxical, but, for example, we will get the same result, for example, in the case of atom while being allocated with the help not of scattered light but of light emitted by atom itself. If the light is emitted in the form of quantum  $\hbar\omega$ , then atom will receive recoil momentum  $\frac{\hbar}{\lambda}$ , and again the study of atoms position will depend on its velocity changes. In both cases the accuracy of atom position determined with the help of scattered or emitted light equals to the wavelength of the light, and momentum change connected with it will be inversely  $\lambda$ . Increasing the measurements accuracy of particle position, we enlarge the error of definition of its momentum. In the result it is impossible to determine the particle momentum at the exact moment of time, when the position of particle is determined since the momentum of particle sharply changes at that moment. The same considerations would be taken into account at determining of velocity also, that resulted in famous Heisenberg relations.

The following philosophical problem appears: is it possible, in principle, to observe any phenomenon without changing it or interfering in it? This problem is no doubt quite old and banal. Anybody agrees that, for example, measuring the electric potential of any matter should change this potential to a certain degree. Any innovations of that measuring apparatus have dealt mainly with tendency to enlarge voltmeter internal resistance and with unachievable idea to make it equal to infinity. Every experimentalist has learned to take into account such non-ideal characteristics of instruments in the process of measuring. And nobody was thrown into confusion with that.

It was proudly announced at the outset of quantum theory that micro-particle

does not have at the same moment of time the exact values of co-ordinate and momentum and their values are connected by relation:

$$\Delta x \cdot \Delta p \geq h, (*)$$

and that statement as well as inequality was called as corresponding to nature of micro-worlds objects and quite not caused by lack of appropriate measuring instruments. But the following question may be put: what will happen if within future decades indirect methods possible to use for measuring purposes will be opened? Nowadays even the problem of mass spectrum is infinitely far from solution and nobody can say whether there is or not any indirect methods. Who is able to foreseen the future?

Shortly after that another relation was derived, viz. between energy and moment of time, when that energy being measured:

$$\Delta E \cdot \Delta t \geq \hbar$$

That relation appeared in great number of books due to intellectual inertia of some authors. And only much later the investigators made out that such relation did not exist within strict quantum mechanics as well as the following relation

$$t \cdot \hat{H} - \hat{H} \cdot t = i\hbar$$

did not exist.

On the other hand, the operator relation

$$x \cdot \hat{p}_x - \hat{p}_x \cdot x = i\hbar$$

---

(\*) It is the simplified form of the Heisenberg relation. The strict relation is expressed by the dispersions of the errors  $\Delta x, \Delta p$ .

exists and results in uncertainty relation for the coordinate and momentum.

To get the uncertainty relation for the energy and time, the energy operator  $i\hbar \frac{\partial}{\partial t}$  should be similar to momentum operator  $-i\hbar \frac{\partial}{\partial x}$  for  $p_x$ . But in reality, according to strict quantum theory, the energy operator  $\hat{H}$  is an operational relation for momentum and coordinate operators, i.e.

$$\hat{H} \equiv \hat{H}(\hat{p}_x, \hat{p}_y, \hat{p}_z, x, y, z)$$

So, the energy within strict quantum theory is a quantity with quite definite value at given moment of time, but time  $t$  in contrast to coordinates  $x, y, z$ , is not an operator. That is why time plays in quantum theory quite special role.

N. Bohr have obtained the same relation after manipulating with wave packets of de Broglie waves (creating a particle from these waves packets), but he had carefully forgotten that these wave packets were spreading. To put it mildly that approach is not quite correct. More over the principle of complementarity offered by Bohr ad hoc, forbade the constructing any speculative models of particle's motion. Since that the main task of the physics became the search of mathematical expressions to be set in one experimental data to obtain the other by computations. According to it, the lack of picture in images and motions within quantum physics is not the object of anxiety.

We would like to rehabilitate the strict standard quantum theory and notice once again that, according to it, the uncertainty relation is obtained as the relation between canonically conjugate additional dynamic variables, and we have nothing to say against. In the essence, the corpuscular – wave dualism became the winner. As we can see now, the uncertainty relation is without any doubts valid

but methods used at first for its obtaining were not totally adequate.

UQT overcomes the situation quite easily [172, 183, 200, 201]. As far as the particle (wave packet) is periodically appearing and vanishing at de Broglie wave length (more precisely, the packet disappears twice, and the probability of its detecting is sufficiently big in maximum region only) the position of such a packet may be detected with error

$$\Delta x \geq \frac{\lambda}{2}$$

and then

$$\Delta x \cdot P \geq \frac{h}{2}.$$

As at measuring of momentum module is inevitable the error  $\Delta P = 2P$ , then we have following inequality:

$$\Delta x \cdot \Delta P \geq h$$

The statements of standard quantum mechanics that particles do not have a trajectory become more understandable. Of course, there is a lot of truth in those words. First, it is possible to say so about intermittent (dotted) motion of the particle with oscillating charge. Second, any packet (particle) is able during its motion to split into few parts. Each of that parts being summed with vacuum fluctuation may results, in principle, in few new particles. Or vice-versa the broken particle may vanish at all and contribute to general fluctuating chaos of the vacuum. But in any case it is better to have more clear idea of particle concrete motion than operate with generally accepted nowadays-obscure sentence about lack of trajectory.

If we turn a retrospective look into all philosophic-physical mess dealing with

uncertainty relation, it is impossible to throw off the idea that phenomenon of social-scientific idea were predicted by V. I. Lenin in his work “Materialism and empirical criticism” long before quantum mess:

*“The really important cognitive-theoretic question, dividing philosophical tendencies, is not a degree of accuracy that our description of causality have achieved and whether these descriptions are expresses in fine mathematical formulation, but whether objective rule of nature is the source of our knowledge of these connections, or it is the property of our mind, its ability to cognize the known a priori truth inherent to it...”* (italics is our).

The uncertainty relation is usually used for justification of non-determinism of quantum theory because it makes nonsensical application of Laplacian determinism to microcosm phenomenon. First of all, uncertainty relation itself has no connection with the question of truth or falsity of determinism because it only reveals the sense of quantum state concept, but neither truth nor falsity of determinism. Second, uncertainty relation really makes nonsensical the application of Laplacian determinism to microcosm phenomena. Actually, if there are no definite values of coordinate and momentum, then in this case their simultaneous definite values cannot be predicted in future too. That is a philosophical reason. Physical reason is that random non-foreseen vacuum fluctuations may change both particle’s coordinate and momentum and agrees, in essence, with philosophical reason. UQT show that determinism in physics has not the Laplacian form only and, in general, has not only a form inherent to classical physics.

The whole preceding science was based on classical description of objects without taking into consideration material character of the observation process. In other words it was the description of objects or processes “in itself”. Quantum science has assigned some limit of such understanding, and although UQT allows describing hypothetically the behavior of quantum objects in “images and

motions” there is now either above mentioned hypothetical researchers or their hypothetical experimental devices, and we will have to be content with experimental data obtained with the help of macro-devices.

The principle of complementarity introduced by N. Bohr cannot be explained so easily as it is in the case of uncertainty relation, because it is a set of some philosophical discourses with marks of previous years fight between materialism (it is also called Marxism-Leninism) and other philosophical trends. We would like to isolate ourselves from any politics; the authors do not sympathize any politics and philosophical brawls, and try never to participate in it. Nevertheless, there are objective laws that will not be changed even authors and readers disappear, and politicians declare the collapse of materialism and of the said laws. As UQT is able to show many “intimate” sides of quantum behavior and to give the sufficient interpretation of existing quantum processes, the result is quite simple: materialism is gained.

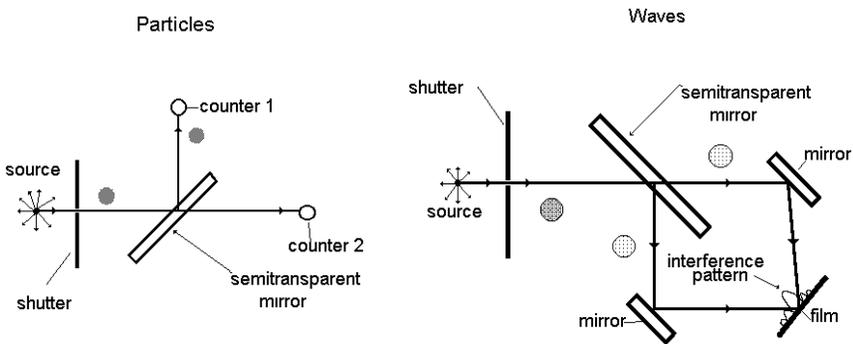
Let us consider in details the principle of complementarity. It is hard to disjoint it from uncertainty relation. Even the origin of its name came from ordinary mechanics, where operators non-commutating with each other correspond to complementary quantities. As we have seen above the uncertainty relation descends from that also. Nevertheless, a lot of philosophical explanations had been appeared which Bohr even had not suspected of. The principle of complementarity can be stated quite popular as follows:

1. A quantum object is extremely complicated formation, not quite easily understood yet, and it corpuscular and wave characteristics are absolutely unlike and only supplement each other. We can draw rough analogy: maps of Eastern and Western hemispheres, men’ photos in full and half face and so on.

2. There are two classes of experimental devices. With the help of one we can measure the coordinate, the energy and the momentum – the attributes of a particle. With other, while observing the processes of interference or diffraction, we can measure the wavelength. At any measuring (in cases of small energies) particle “is lost” or its parameters change radically in the result of macro situation effect. It is called as uncontrolled effect that is why it is impossible to measure at the same moment of time corpuscular and wave parameters.
3. We should not ask Nature questions that will not be experimentally answered.
4. It is not necessary to make attempts in constructing the quantum pictures in images and motions as it was within before-quantum science. It is quite enough to be able to solve and analyze different quantum equations mathematically and to apply the new rules derived within quantum mechanics.

The attitude of Paul Langevin to the last two items was as to something disgusting and he called the principle of complementarity as “*intellectual debauchery*”.

The other numerous statements are based on variants of uncertainty relation.



**Fig. 2.13.1** Experiments with individual photons on semitransparent mirror.

There were many physical and philosophical discussions about photon behavior at semitransparent mirror (Fig. 2.13.1). With the help of complementarity principle it was analyzed in what flux (reflected or penetrated) the photon is located while the interference of penetrated or reflected flux is observed and how it correlate with the number of particles to be appeared in penetrated and reflected fluxes. When the flux of particles falling down on the translucent mirror one after another was observed with big exposition, then the interference picture became visible. It contradicts the fact that the particles was detected either in penetrated or reflected flux, and it is incomprehensible how could the interference picture arise. If the particle remains in reflected flux, then it could not been observed in the passed flux, and it is impossible to understand what and with what would interfere. The observed facts of rare simultaneous signals of two particle counters were explained by random appearance of two photons “nearby”, and one of them has penetrated the mirror and the other – was reflected. There were some reasons due to observations of induced radiation (the main principle the lasers are based on). There were made quite enough different experimental variations at that matter [31-41]. We should note that they do not contradict the ideas developed within UQT.

Of course not only the processes of splitting cause the phenomena of

interference and diffraction. It is shown in section 2.11 that even indivisible particle described by equation with oscillating charge while spreading is able to show the behavior having seemingly a wave character. All these processes look very knotty.

N. Bohr has offered well-known interpretation of that phenomenon from the principle of complementarity viewpoint. We shall remind it shortly. The particles' flow falling down at the mirror is described by wave function (i.e. by the amplitude of probability). The particle after hitting at translucent mirror is, so to say, in a potency state: the particle may belong to penetrate or to reflected flux, it may be appeared (detected) and maybe not. Namely, that potency is interfering, i.e. possibility of particle's location here or there. These potential possibilities become actual at the finish of object and device interaction only. And though probabilities are referred to potential-possible, i.e. to non-finished experiment, but statistics based on these probabilities is a statistics of realized interactions, i.e. of finished experiments. But if an experimental device would be created being able to follow the destiny of individual particle and to detect to what flux (penetrated or reflected) the particle belong, then the particle would be absorbed or its parameters were changed at such a value that we would not be able to speak about its participation in interference process. If this process is studied, then it is impossible without violation of interference process to detect the flux, where the photon is located. Either one thing or another, they cannot exist together.

We should note, - it is worthy of astonishment that N. Bohr was able to imagine that principle and interpretation, because it turned out that if one follows strictly the prescribed principles and rules, then the right results are obtained and no contradictions arise.

All paradoxes were eliminated by simple prohibition to think about it. It stimulated a great philosophical discussion but physicists did not pay attention at.

And they were right since that discussion took the form of some talks resulted in nothing, but orthodox quantum interpretation answered every physical question to be asked within new unusual game rules and served as perfect instrument of knowledge. Nevertheless for any thinking researcher the question whether it is true is raised always. Why we could not even imagine that particle has exact values of momentum and coordinate and follow its dynamics in details? Why we could not study with any indirect methods the concrete sides of particle motion (as it takes place in other sciences)?

Absolutely new philosophical problems about “free will” and even about the existence of particles in connection with probability interpretation of wave function have appeared. Religion was also admixed due to A. Eddington.

There was quite solitary the question about the cause of quantum mechanics statistical character. In connection with that the words of A. Einstein are quoted especially frequently about his unbelief in “God is playing cards”. There are so many different speculations about that. But the main is that statistical interpretation does not belong to quantum mechanics instrument and does not result from it but simply postulates. That is not so within our UQT and the probability of phenomena appears due to inner content of this theory, and, as we hope, the question about how “*God plays cards*” has disappeared for most part of our readers at the moment of reading these words.

The authors are sure that all additional philosophical quantum-mechanical images of the nature will be crushed down in the nearest future and UQT will gain, and the above mentioned problems will surprise future generation as well as now we are amazed at ancient opinions about three elephants and three whales supporting our Earth. It is astonishing but even these quite naïve ideas had relaxed or rather lulled humanity mind during very long time.